

Lesson Plan

Module:	Patterns		
Teaching Hours:	6 x 40-minutes		
Grade Level/Age Range:	Grade 6		
Brief Description:	The module engages students with growing patterns. Students engage in identifying and representing growing patterns, in finding recursive and functional relations.		
Design Principles:	Inquiry		
	Situatedness		
	Digital tools		
	Embodiment		
	<ul style="list-style-type: none"> • Meaningful: Build on students' intuitive knowledge and daily life experiences with real-life scenarios • Embodiment: Perceptual-motor (action-perception) experiences with noticing the covariation and correspondence relation by grounding the understanding on concrete actions about how the pattern grows • Inquiry based learning: explore recursive and functional relations • Digital: tablet devices equipped with appropriate apps • Didactical phenomenology / situatedness: the covariation and correspondence relations are recorded, tabularized and mathematized 		
Functional Thinking:	Input – Output		
	Covariation		
	Correspondence		
	Object		
Learning Goals:	<ul style="list-style-type: none"> ✓ Identify growing and repeating patterns ✓ Represent and describe growing patterns using words, table, graph ✓ Extend growing patterns using different modes of thinking ✓ Identify covariation and correspondence relations in growing patterns ✓ Express the relations (verbally/symbolically) and generalize 		

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Activities

Explorations

Exploration 1.

Students explore the structure of a growing pattern, that of Human Pyramid, and then attempt to construct a bigger pyramid (not necessarily the “next” one). The teacher selects pairs of students to present their work.

Useful questions: How many people are needed to construct the pyramid? How many people are needed at the base of the pyramid? How many people does it take to build the next biggest pyramid? Could we create a pyramid with 16 people?

Suggested tools/materials: Video

Estimated duration: 15 minutes

Exploration 2.

Students work in pairs on the “Slider & Figures” app. Students are asked to explore the app and drag the cursor to create pyramids of various sizes. Then, they are asked to find how many squares are needed to build the next pyramid (Pyramid 12) and explain how they can find the number of squares when they know the number of the pyramid.

Useful questions: How many squares are needed to build Pyramid 12, Pyramid 13 and Pyramid 14? What does it change and what does it stay the same every time? How many more squares are needed each time? How can we calculate the squares needed for Pyramid 12 without measuring them?

Suggested tools/materials: GeoGebra App

Estimated duration: 15 minutes

Activities

Activity 1.

Students are invited to explore the application. They can vary the number of grey and green squares in order to explore how the pattern develops. In this app, each subsequent term is equal to the sum of the previous two terms minus one.

Suggested tools/materials: GeoGebra App

Estimated duration: 10 minutes

Activity 2.

Students work in pairs on their tablet devices. Students are asked to explore the app, find out how it works, drag the slider to get different values of grey squares. The same recursive rule applies when changing the number of grey squares. Students could engage in making a hypothesis about the pattern rule and then check it using the option “Next figures”.

Useful questions: How does the pattern continue? Why is it a pattern? How would you describe the pattern to someone who has never seen it? Find different ways to describe the pattern. How many more squares each next figure have?

Furthermore, students are anticipated to engage in finding the correspondence relation between the Figure number and the number of squares using different representational tools (words, tables, symbols). Depending on students’ level, the teacher may select the level of difficulty of the correspondence relation. For example, when Number of grey squares=1, the rule is Figure $n = 2n - 1$. When Number of grey squares=2, the rule is Figure $n = 2n$, and when Number of grey squares=3, the rule is Figure $n = 2n + 1$.

Suggested tools/materials: Tablet devices, Geogebra App

Estimated duration: 30 minutes

Activity 3.

The task is about Chris who creates designs for textiles. The growing pattern involves three quantities: the number of black squares, the number of grey squares and the total number of squares.

Questions (a), (b) and (c) engage students in identifying and describing the structure of the pattern while Question (d) engages students in expressing the recursive rule in rather general terms since students are asked how many more squares each consecutive section would have. Afterwards, in Question (e), students are gradually encouraged to find the functional relation between the number of black squares and the total number of squares. Students are asked to work forward from the number of black squares to find the total number of squares and also to work backwards from the total number of squares to the number of black squares. In Question (f), the general rule is given and students are asked to share their reasoning why this rule holds true.

Students could work independently for Questions (a)-(e), then share with their answers and discuss Question (f). Afterwards, in whole class discussion, students are encouraged to explain how they filled in the tables and why Chris’ rule works.

Suggested tools/materials: Grid paper

Estimated duration: 30 minutes

Activity 4.

Students are asked to use the app and make their own patterns using the pattern blocks digitally. Their patterns could be repetitive or growing (depending on their level). Also, the teacher could set the rule of the pattern (e.g., every time I add 2, the general rule is $2n-1$).

Suggested tools/materials: Tablet, App

Estimated duration: 10 minutes

Activity 5.

Students engage with a growing geometric pattern. This task relies on students' experiences with the previous tasks and it provides an opportunity for further practice.

(Questions (b)-(h) could be adapted for other patterns, if further practice is needed)

Suggested tools/materials: Grid paper, cubes

Estimated duration: 40 minutes

Extension Activities:

This section presents task that rely on students' experiences with the previous activities and intends to provide further opportunities for practice.

In Activity 1, students are expected to engage with a pattern from nature. Students are asked to find the number of hexagons for the 3rd step of the honeycomb. Then, students organise the information into a table, in which the correspondence between the step number and the number of hexagons starts to become more visible. Students are also asked to express how many more hexagons they would need for the 4th step in order to identify how the pattern changes as it grows. Students work in groups and selected groups share their work with the whole class.

In Activity 2, students work on another app where they can vary the number of grey squares. They are asked to find the pattern rule (recursive rule) and check their answer by selecting 'Next figures'. (When Number of grey squares=1, then Fibonacci sequence is shown).

In Activity 3, students work again with the Fibonacci sequence. Connections are expected to be made with the previous application (number of grey squares=1). Students are asked to describe the rule, explain how the sequence continues, and describe the graphical representation of the Fibonacci sequence. Additionally, students are engaged in determining how the Fibonacci sequence appears on a shell using the given representation. Students can also work on constructing the Fibonacci sequence by colouring squares on grid paper.

In Activity 4, students are asked to create their own growing pattern without any restriction and in Activity 5, with the restriction of following a given rule. For Activity 4, students need to think of how the first figures would look like, how the pattern would grow in a systematic way. For Activity 5, students would need to interpret the rule, understand what $4n$ and $+4$ stand for. The teacher could provide a hint, if needed, to look for patterns that share the same.

In Activity 6, students engage with a growing number pattern. Students are anticipated to work from the recursive rule towards finding a functional rule that corresponds the place with the number and express it verbally and symbolically.

Suggested tools/materials: App, Grid paper

Estimated duration: 80 minutes

Assessment

1. Select the ones that show patterns. Explain.

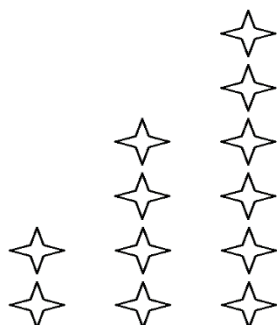
(a) 0, 2, 4, 7, 9, 11, 13, ...

(b) 3, 6, 12, 24, 48, 96, ...

(c)



(d)



2. Zoe is creating the following pattern.

(a) How many squares would Figure 4 have?

(b) How many more squares would each next figure have?



Figure 1

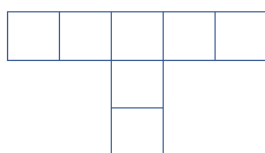


Figure 2

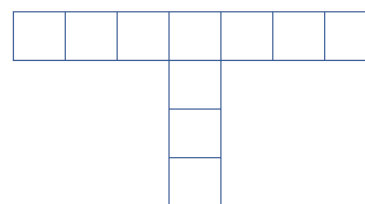


Figure 3

3. A pattern begins with number 5 and repeatedly adds 4. What are the first five terms of this pattern?

4. Follow the instructions to complete the pattern in the box below.

“Your first step is to start with number 1. Your second step is to add 3. Then, you will add 3 to each term to get to the next term”.

(a) Complete the following table.

Step Number	Pattern Number
1	1
2	
3	
4	
5	

(b) Create a graph (using grid paper or dynamic geometry software) based on the table above using as coordinate points the step number and the pattern number.

(c) Use the graph to find the pattern number for step number 12.

5. Kai is constructing the following pattern.

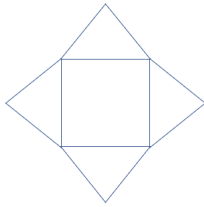


Figure 1

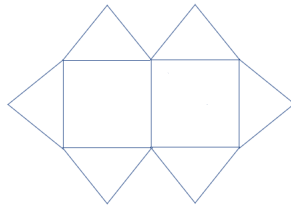


Figure 2

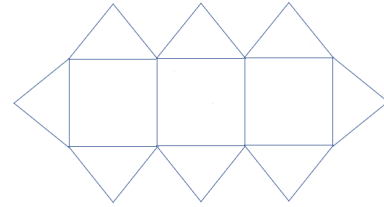


Figure 3

(a) How many triangles would Figure 12 have?

(b) How many triangles would Figure n have?

Digital Tools:

Exploration 1:

<https://www.youtube.com/watch?v=t179ZcUdCOA&t=241s>

Exploration 2:

<https://www.geogebra.org/m/vcypf5kn>



Activity 1:

<https://www.geogebra.org/m/uspig538>



Activity 2:

<https://www.geogebra.org/m/rusymz3d>



Activity 4:

[Pattern Shapes by The Math Learning Center](#)