



Function definition and representations

Solve the following four tasks. Reflect on the method of solving and what is the aim of them.

TASK 1. Does a function exist which domain is interval $(0,5)$ and the set of values is $[2,5]$? Justify your answer.

TASK 2. Does a function exist which domain is $\{1,2,3\}$ and the set of values is $\{1,2\}$? Justify your answer.

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TASK 3. Does a function exist which for any real x, y numbers fulfils the requirement:

$$f(x + y) = f(x) \cdot f(y) . \text{ Justify your answer.}$$

TASK 4. If we substitute 1 for x in the expression $ax^2 + bx + c$ we get a positive number, substituting 6 we get a negative number.

How many solutions does the equation $ax^2 + bx + c = 0$ have? Justify your answer.

Source of tasks:

- Sajka, M. (2019). *Pojęcie funkcji. Wiedza przedmiotowa nauczyciela matematyki*. Wydawnictwo Naukowe Uniwersytetu Pedagogicznego, DOI 10.24917/9788380841048, p. 102, 107-108. (Tasks 1-3)
- Even, R. (1990). Subject Matter Knowledge for Teaching and the Case of Functions, *Educational Studies in Mathematics*, Vol. 21, No. 6. (Dec., 1990), pp. 521-544. (Task 4)



Function definition and representations – Handout 2

Read the text below and the examples from the school textbook and the task book. What do you discover?

1

According to Sajka (2019, pp. 15-16) :

“In mathematics, there are two fundamentally different approaches to defining the concept of function, two contrasting definitions of the notion of function formulated by Peano (1911) and Hausdorff (1914):

1. According to Peano (1911), a relation is a certain set of ordered pairs, and in turn a function is a certain special type of relation in which if the pair (x, y) and (x, z) are pairs belonging to the relation, then $y = z$.

2. Hausdorff (1914) first defined the product of any sets A, B as the set of all ordered pairs $p = (a, b)$, where $a \in A$ and $b \in B$, and then wrote:

- (...) we shall consider a certain set P of such pairs, having, namely, the property that each element a of A occurs in the first place in one and only one pair p of P . Each element a thus determines one and only one element b , the very one with which in the pair $p = (a, b)$ it is connected; this element determined by a , dependent on a , assigned to a , we denote by $b = f(a)$ and we say that by this in A (i.e. for all elements from A) a certain unambiguous function has been defined. We regard two such functions $f(a), f'(a)$ as equal if and only if the associated sets of pairs P, P' are equal, so that for every a there is $f(a) = f'(a)$ (Hausdorff, 1914, p. 33)”

Source:

- Hausdorff, F. 1914, *Grundziige der Mengenlehre*, Leipzig.
- Peano, G. (1911). Sulla definizione di funzione, *Atti della Reale Accademia dei Lincei, Serie 5a, Classe di scienze fisiche, matematiche e naturali* 20, 3-5.
- Sajka, M. (2019). *Pojęcie funkcji. Wiedza przedmiotowa nauczyciela matematyki*. Wydawnictwo Naukowe Uniwersytetu Pedagogicznego, DOI 10.24917/9788380841048.

2

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8. Function and its properties

Ways of describing functions

The most common ways we use to describe functions are:

- (a) verbal description
- (b) a table
- (c) a graph
- (d) a set of ordered pairs
- (e) a formula
- (f) a graph

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0. **8.10.** Dana jest funkcja, przedstawiona w postaci zbioru par uporządkowanych. Narysuj wykres tej funkcji.

- a) $\{(-2, -1), (-1, 0), (0, 1), (1, 2), (2, 3), (3, 4)\}$
 b) $\{(-4, 3), (-3, 4), (-2, 0), (-1, 1), (0, 3), (2, 4)\}$
 c) $\{(-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1)\}$
 d) $\{(-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)\}$

A function is given, represented as a set of ordered pairs. Draw the graph of this function.

1. **8.11.** Dana jest funkcja, opisana za pomocą zbioru par uporządkowanych. Podaj wzór tej funkcji.

- a) $\{(-2, 3), (-1, 4), (0, 5), (1, 6)\}$
 b) $\{(-4, 8), (-3, 6), (1, -2), (2, -4), (3, -6), (4, -8)\}$
 c) $\left\{\left(-\frac{1}{4}, -\frac{1}{64}\right), \left(-\frac{1}{3}, -\frac{1}{27}\right), \left(-\frac{1}{2}, -\frac{1}{8}\right), (0, 0), (2, 8), (3, 27), (4, 64)\right\}$
 d) $\{(-8, 5), (-3, 5), \left(-\frac{1}{2}, 5\right), (3, 5), (11, 5)\}$

A function is given, represented as a set of ordered pairs. Give the formula of this function.

2. **8.12.** Dana jest funkcja, opisana za pomocą zbioru par uporządkowanych: $\{(1, 3), (-2, 7), (7, 4), (0, 0), (8, 1)\}$.

- a) Podaj wartość funkcji dla argumentu 7.
 b) Podaj argument funkcji, dla którego wartość funkcji wynosi 1.

A function is given, represented as a set of ordered pairs.
 a) Calculate the value for the argument 7.
 b) Calculate the argument for which the value of the function equals 1

(...) Draw the graph of the function

- 8.14.** Dana jest funkcja, opisana za pomocą zbioru par uporządkowanych. Narysuj wykres tej funkcji.

- a) $\{(x, y): |x + 3| \geq 1 \text{ i } y = -x\}$ b) $\{(x, y): |x - 2| < 2 \text{ i } y = x - 1\}$

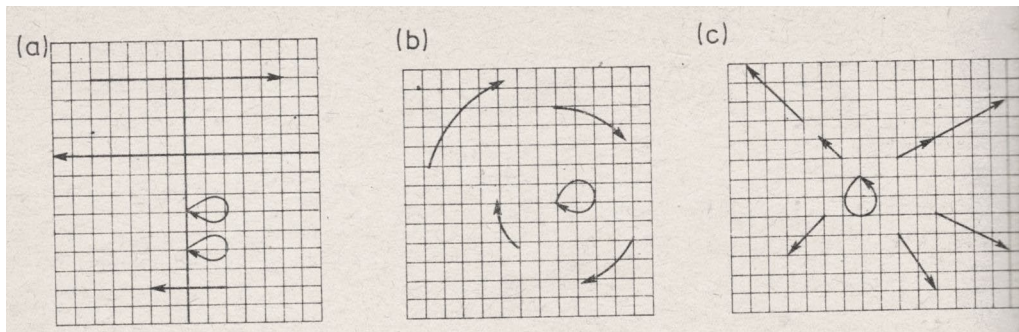
Source:

(2) Kurczab, M., Kurczab, E., Świda E. (2015). Textbook: „Mathematics 1 to secondary and technical schools”, Wydanie IV, Warszawa, p. 286;

(3) Kurczab, M., Kurczab, E., Świda E. (2014)., Matematyka 1, Zbiór zadań do liceów i techników, [Mathematics 1 - task book to secondary and technical schools], Wydanie III, Warszawa, 2014, p.203.

Function definition and representations

Reflect on these graphs:



Source: Turnau, S. (1990). Wykłady o nauczaniu matematyki [Lectures on the teaching of mathematics], WSiP, Warszawa, p. 178

- Which transformation is represented by the following graphs?
- How do we name such graphs?
- Are the transformations presented in this way functions?

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Activity inquiry-based learning

In the following, you see three different approaches to introduce the concept of powers.

1. Compare the three approaches. To what extent do the approaches incorporate students' prior knowledge?
2. Which example would you choose? Provide reasons.

1

According to a legend, long ago in one of the kingdoms of ancient India there was a powerful and rich emperor named Velchib. A Brahmin priest, named Sissa, invented chess and offered it as a present to the emperor, a chess. The emperor was so impressed and excited with the present to the emperor that he decided to offer him a gift. Velchib asked Sissa what present he wanted.



Sissa thought for a moment and replied "I want two grains of wheat in the first square, four in the second, eight in the third and so on..."

The emperor was puzzled and angry about the cheap gift that Sissa had asked for and ordered his storekeepers to give him the wheat he wanted. However, as things turned out he could not deliver his promise.

✓ Why couldn't the emperor deliver his promise?

Fill in the table:

Square	Number of wheat grains	Result
1	2	2
2	$2 \cdot 2$	4
3	$2 \cdot 2 \cdot 2$	
4		
⋮		
8		
10		
⋮		
32		
⋮		
64		

✓ Explain your strategy

To produce this huge quantity of grains, which is actually a 20 - digit number, one has to plant the whole Earth 76 times!

It is said that the emperor, in order to avoid the insult for not keeping his promise, he was consulted by his advisors to ask Sissa to count all the grains. Something that would take forever!

2

Powers & Exponents

Powers can be used to show repeated multiplication of the same number.

$$\text{Base} \rightarrow 2^{\text{Exponent}} = 2 \times 2 \times 2$$

Power

This is read as "two to the power of three"

3

Use your calculator to complete the following table

	Result		Result
$2 \cdot 2$		2^2	
$2 \cdot 2 \cdot 2$		2^3	
$2 \cdot 2 \cdot 2 \cdot 2$		2^4	
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$		2^5	
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$		2^6	

(a) What do you observe?

(b) How can we express repeated multiplication of the same number? Provide examples.

(Athanasiou et al., 2016a, p. 47f & 2016b, p. 17)

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Videos for teacher course

Learning goals:	Teachers can reflect on different strategies they can use when reacting on students' incorrect answers
Suggestion/questions for discussion.	<ul style="list-style-type: none"> ○ Introduce the context necessary for video understanding – especially the task ○ Individually: write down what you've seen in the video as precisely as possible ○ In small groups: interpret what teacher and student(s) were thinking and think about possible consequences (if this was the most common reaction on students' error)
Video 1	
Video reference name:	Marbles 2 IV
Web link:	
Task/learning environment in video:	Marbles,
Web link to task description:	
Brief Description:	<p>Students are trying to fix the error they found in their solution. They expect the values to lie on one straight line. However, one of the values does not fit.</p> <p>A researcher (in the role of teacher) asks questions to reveal their understanding of the error.</p>
Transcription in original language	<p>1 00:00:02,000 --> 00:00:04,610 No tak to podrž, hej? Lebo ja lejem.</p> <p>2 00:00:10,000 --> 00:00:12,000 Čo robíte, dievčatá?</p> <p>3 00:00:13,000 --> 00:00:16,000 Sekli sme sa, tak to musíme urobiť ešte raz.</p> <p>4 00:00:20,000 --> 00:00:23,000 Ako ste zistili, že ste sa sekli?</p> <p>5 00:00:23,300 --> 00:00:27,000 Nevyšla nám na grafe tá čiara presne.</p> <p>6 00:00:30,580 --> 00:00:31,580</p>

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...nenapadlo.

7

00:00:31,580 --> 00:00:32,580

A čo vás malo napadnúť?

8

00:00:32,580 --> 00:00:33,580

Že to je krivá čiara.

9

00:00:37,580 --> 00:00:39,580

To je presne na 160 tu.

10

00:00:40,080 --> 00:00:41,580

Čierna mikina: A ako to môže byť toto?

11

00:00:46,580 --> 00:00:50,080

Čierna mikina: Počkaj, ale nie.

Červená a sivá: Áno!

12

00:00:50,080 --> 00:00:54,580

Čierna mikina: Ty nie prvú meníš. Ty druhú meníš.

13

00:00:49,880 --> 00:00:55,580

Červená mikina: Druhú meníš!

Čierna mikina: No!

14

00:00:56,880 --> 00:00:59,580

Sivá mikina: No však druhú.

Čierna mikina: No však prvá :)

15

00:01:12,580 --> 00:01:13,580

A prečo to musí byť čiara?

16

00:01:15,580 --> 00:01:16,580

No? Nataša?

17

00:01:16,880 --> 00:01:26,580

Ja neviem, akože tak mala by to byť rovná čiara, keď je to graf. Nie?

18

00:01:26,880 --> 00:01:28,580

Nataša, vo fyzike sme robili také grafy, čo si ...

19

00:01:28,880 --> 00:01:32,580

Tak si to mala ty povedať. Ja to neviem.

Transcription in English

1

00:00:02,000 --> 00:00:04,610

Hold it, OK? I'm pouring.

2

00:00:10,000 --> 00:00:12,000

What are you doing girls?
3
00:00:13,000 --> 00:00:16,000
We made a mistake, so we have to do it again.

4
00:00:20,000 --> 00:00:23,000
How did you find out you were wrong?

5
00:00:23,300 --> 00:00:27,000
We didn't get the straight line exactly.

6
00:00:30,580 --> 00:00:31,580
It didn't come to our minds.

7
00:00:31,580 --> 00:00:32,580
What exactly you mean?

8
00:00:32,580 --> 00:00:33,580
That it's a crooked line.

9
00:00:37,580 --> 00:00:39,580
That's right here, at 160.

10
00:00:40,080 --> 00:00:41,580
Black sweatshirt: And how can this be this way?

11
00:00:46,580 --> 00:00:50,080
Black sweatshirt: Wait, no.
Red and Gray sweatshirt: Yes!

12
00:00:50,080 --> 00:00:54,580
Black sweatshirt: You do not change the first one. You change the second one.

13
00:00:49,880 --> 00:00:55,580
Red sweatshirt: You change the second one!
Black sweatshirt: Well!

14
00:00:56,880 --> 00:00:59,580
Gray sweatshirt: But the second one.
Black sweatshirt: But the first one :)

15
00:01:12,580 --> 00:01:13,580
And why does it have to be a straight line?

16
00:01:15,580 --> 00:01:16,580
Well? Natasha?

17
00:01:16,880 --> 00:01:26,580
I don't know. It should be a straight line when it's a graph. Right?

18

	<p>00:01:26,880 --> 00:01:28,580 Nataša, in physics we made such graphs, which you ... 19 00:01:28,880 --> 00:01:32,580 You should have said it. I do not know.</p>
Video 2	
Video reference name:	Marbles 03 IV
Web link:	
Task/learning environment in video:	Marbles – Filling the table for growing number of marbles
Web link to task description:	
Brief Description:	<p>Students firstly ask the researcher (in the role of a teacher in that moment) to solve their problem of initial value – 0 or 150. She lets students discuss things and after, she poses the question.</p> <p>She also notices that students are working with incorrect volume of one marble. She poses questions in such a manner that students observed something is not good.</p>
Transcription in original language	<p>1 00:00:04,001 --> 00:00:08,000 Čiže počet guľočok vo vode je 0, takže objem je 150.</p> <p>2 00:00:08,100 --> 00:00:11,000 Ale ideme od 150 alebo od 0?</p> <p>3 00:00:13,000 --> 00:00:14,000 No?</p> <p>4 00:00:14,100 --> 00:00:16,500 No veď hádžeš to do vody, takže voda tam už musí byť.</p> <p>5 00:00:17,100 --> 00:00:11,000 OK, keby tam bola 0 na začiatku, ako by to vyzeralo?</p> <p>6 00:00:31,000 --> 00:00:34,500 Takže tu musí byť 150 ako prvé.</p> <p>7 00:00:45,500 --> 00:00:48,000 OK, ... teraz skúste mi povedať, čo píšete?</p> <p>8 00:00:49,300 --> 00:00:56,000 No že keď vhodíme guľôčku do vody, tak sa voda zdvihne o 5.</p> <p>9 00:00:56,500 --> 00:00:58,500 OK, takže jedna guľôčka má objem</p> <p>10</p>

	<p>00:00:58,500 --> 00:00:59,000 5. 11 00:00:59,000 --> 00:01:01,000 5? Tak ste sa dohodli pred tým? 12 00:01:01,300 --> 00:01:02,000 Áno. 13 00:01:08,100 --> 00:01:10,000 ...budeme písať: Vždy o 5 mililitrov. 14 00:01:22,100 --> 00:01:18,000 Prečo o 5 mililitrov? 15 00:01:25,100 --> 00:01:28,000 Prečo o 5 mililitrov? Na základe čoho ste určili tých 5 mililitrov? 16 00:01:28,500 --> 00:01:32,000 Lebo sme vhodili jednu guľôčku do vody. 17 00:01:32,500 --> 00:01:37,000 A sedí vám to s tým, čo ste robili pred tým? Že keby ste vhodili 5, tak o koľko by sa dvihla hladina? 18 00:01:37,500 --> 00:01:38,500 25. 19 00:01:40,000 --> 00:01:41,500 A to neseďí. 20 00:01:43,000 --> 00:01:47,500 Keď sme vhodili 5, tak sa nám zdvihla o 10. 21 00:01:49,000 --> 00:01:50,500 Neseďí to. 22 00:01:50,500 --> 00:01:52,000 Ale prečo nám to neseďí? 24 00:01:50,500 --> 00:01:52,000 Ja neviem!</p>
<p>Transcription in English</p>	<p>1 00:00:04,001 --> 00:00:08,000 So the number of marbles in the water is 0, so the volume is 150. 2 00:00:08,100 --> 00:00:11,000 But do we start from 150 or from 0? 3 00:00:13,000 --> 00:00:14,000 Well?</p>

4
00:00:14,100 --> 00:00:16,500
Well, you're throwing them into the water, so the water must already be there.

5
00:00:17,100 --> 00:00:11,000
OK, if there was a 0 at the beginning, what would it look like?

6
00:00:31,000 --> 00:00:34,500
So there must be 150 first.

7
00:00:45,500 --> 00:00:48,000
OK, ... now try to tell me what you are writing?

8
00:00:49,300 --> 00:00:56,000
Well, when we throw a marble into water, the water rises by 5.

9
00:00:56,500 --> 00:00:58,500
OK, so one marble has volume ...

10
00:00:58,500 --> 00:00:59,000
5.

11
00:00:59,000 --> 00:01:01,000
5? Do you agreed before?

12
00:01:01,300 --> 00:01:02,000
Yes.

13
00:01:08,100 --> 00:01:10,000
...we will write: Always by 5 milliliters.

14
00:01:22,100 --> 00:01:18,000
Why by 5 milliliters?

15
00:01:25,100 --> 00:01:28,000
Why by 5 milliliters? How did you determine the 5 milliliters?

16
00:01:28,500 --> 00:01:32,000
Because we dropped one marble into the water.

17
00:01:32,500 --> 00:01:37,000
And does it fit with what you were doing before that? That if you dropped 5 marbles, how much would the level rise?

18
00:01:37,500 --> 00:01:38,500
25.

19
00:01:40,000 --> 00:01:41,500
And that doesn't fit.

20

00:01:43,000 --> 00:01:47,500

When we dropped a 5, it raised by 10.

21

00:01:49,000 --> 00:01:50,500

It doesn't fit.

22

00:01:50,500 --> 00:01:52,000

But why does it not fit?

24

00:01:50,500 --> 00:01:52,000

I do not know!