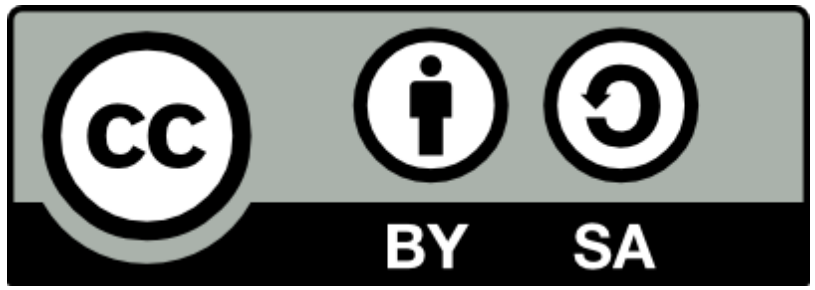

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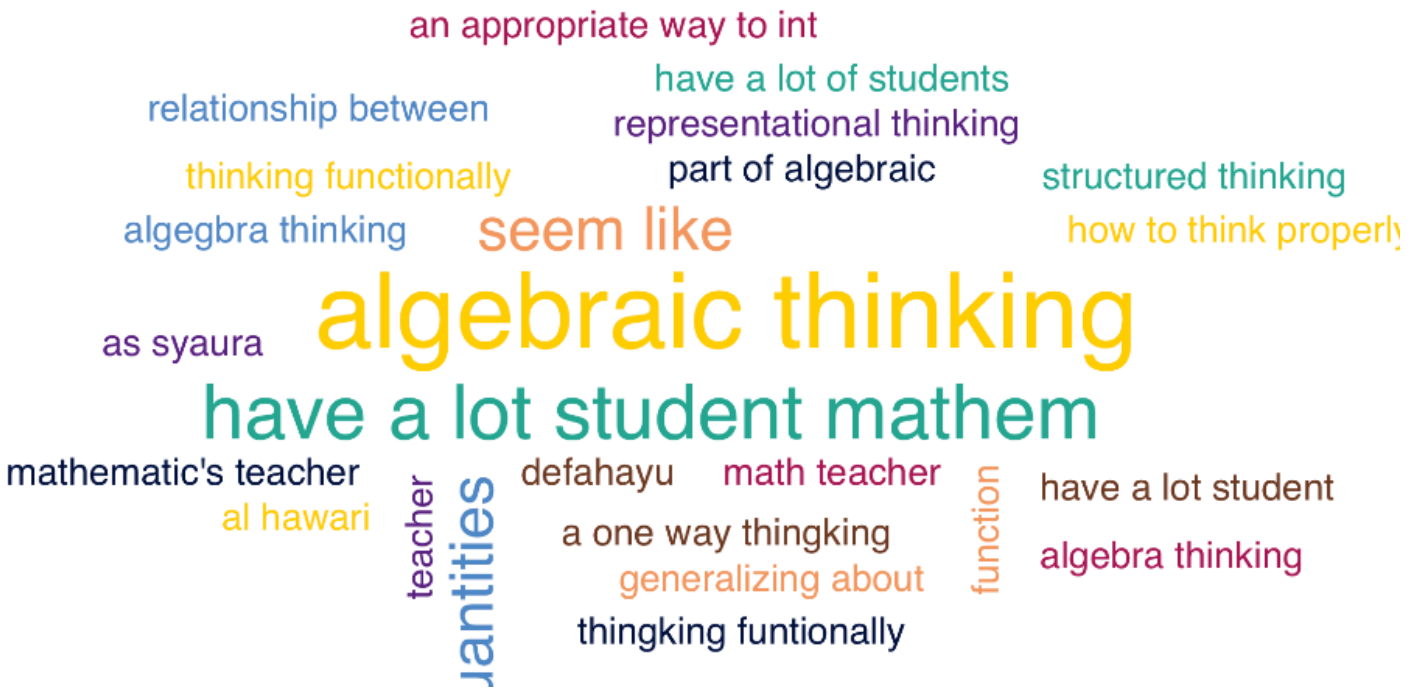
Functional Thinking

What is Functional Thinking?



“A way of thinking in terms of relationships, interdependencies, and change“

“The process of building, describing, and reasoning with and about functions“

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Meaning of Functional Thinking

- Functional relations are involved in many situations of everyday life.
- Understanding the spread of the COVID-19 virus as an exponential function and trying to limit the spread of the virus in the early stages of the pandemic required coordinating different variables and recognizing the relationships between them.
- Functional thinking is involved in all kinds of thinking.
- Functional thinking is the type of thinking that refers to quantitative relationships and develops from people's innate desire to identify connections between quantities and concepts.

(Schotten, 1907)

Education and Functional Thinking

- The Meraner Lehrplan movement in Germany, in 1905, proclaimed the need for "training in functional thinking" as a prerequisite for teaching mathematics in high school.
- This idea became a key slogan of the movement for change in the teaching of mathematics in Germany and other countries at the beginning of the 20th century
- Unifying element in the curriculum.
  - So far, no other concept has generated as much debate about its value in mathematics education (Vohns, 2016).

Definition

Functional thinking has been defined in different ways.

As a point of reference, we adopt the definition of functional thinking as the process of describing, constructing, and reasoning about relations arising from or about functions (Stephens et al., 2017).

Functional Thinking

- Generalization
- Representation of relationship between quantities
- Structure
- Covariances
- Change
- Relationships
- Interpreting qualitative changes to explain relationships
- Leverage relationships to solve problems
- Concept of function = core of functional thinking

Aspects of Functions

- Function as an input-output process
- Function as a dynamic covariance process
- Function as a mapping relation
- Function as a mathematical object

(Pittalis et al., 2020; Doorman et al., 2012)

Function as an input - output process

- Computational nature of function
- Representations: arrow diagram, input-output tables
- It requires no perception of input-output causality
- Examples:
  - Calculation of output value given input value in calculator.
  - Calculation of payment amount for purchase of products depending on the quantity of the items.

Function as a dynamic covariance process

- Understanding Independent and Dependent Variable Covariance – A change in the independent in the definition domain causes a change in the dependent in the value domain.
- A function as two quantities that change simultaneously. (Thompson & Carlson, 2017)
- Representations: table of values, graph
- Examples:
  - Study of the change in distance traveled by an object as a function of time

Function as a mapping relation

- Understanding the relationship between the independent and dependent variable and representing this relationship
- General relationship between the two quantities
- Matching concept
- It leads to the standard definition of the function – Dirichlet definition in high school
- Representations: graphical representations, nomogram
- Examples:
  - Study of relationships between phenomena such as age and risk of contracting the COVID virus

Function as a mathematical object

- Characteristics of a function as a mathematical object and different representations to compare with other mathematical objects.
- Outcome of conclusions for families of functions.
- Higher level process investigation (derivation).
- Representations: graphical representation, symbolic representation of the general formula.
- Example:
  - Comparison of properties of polynomial functions for exponential functions
  - Identifying the characteristics of each family

Description of functional thinking

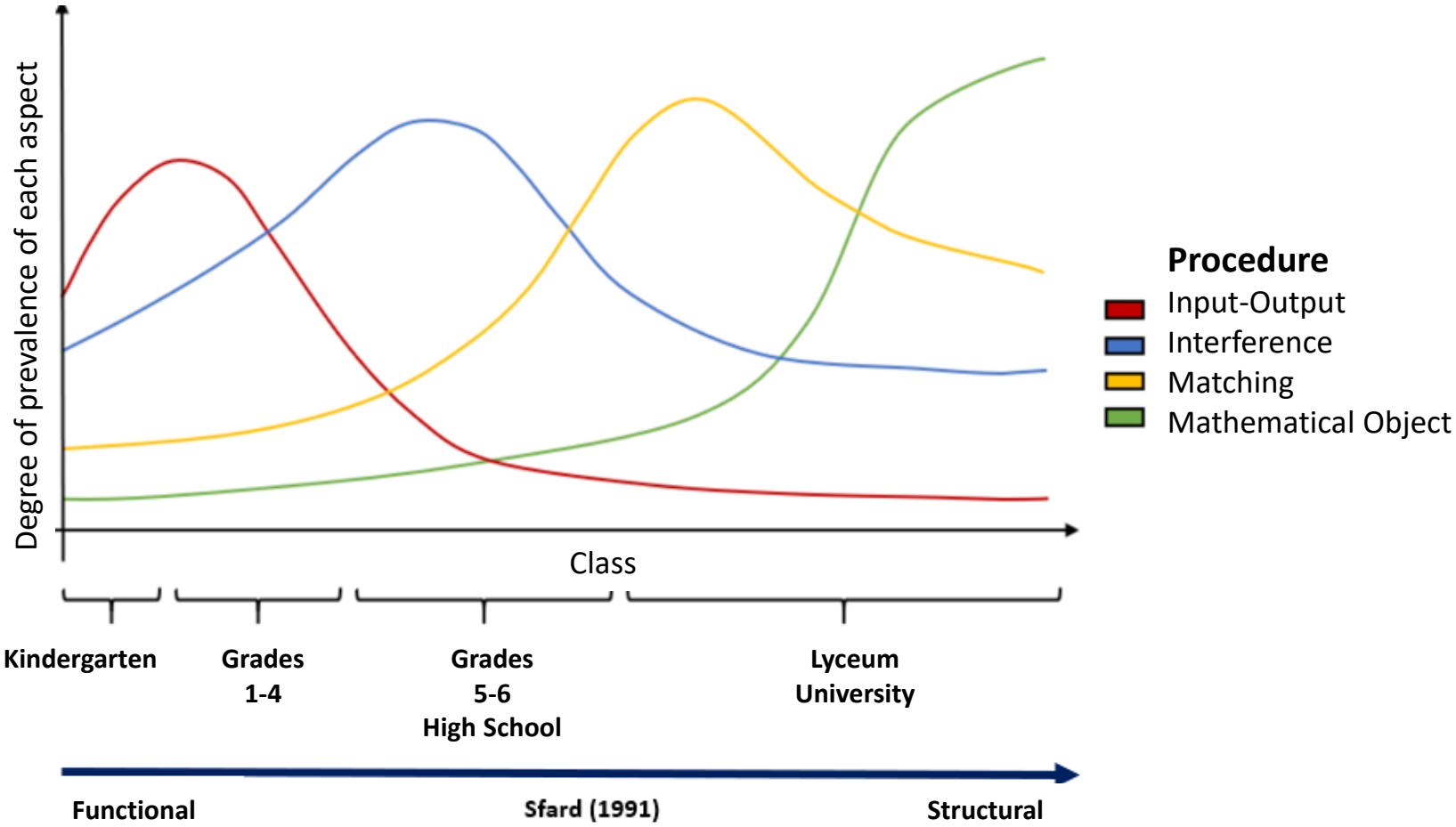
Primary school students

- The kind of thinking that works out the invariant relationship between two quantities/variables that change at the same time.
- It makes it easy to move from handling relationships between specific values to generalizing about relationships that apply to sets of values.
- Identifying and generalizing relationships between simultaneously varying quantities/variables
  - representation of these relationships verbally, symbolically or with another type of representation
  - utilization of these modes of representation in solving problems

For high school students

- This description is gradually enriched with the aspect of the function as a mathematical object and adds to the dimensions of the functional thinking the description, interpretation and exploitation of the properties of the functions according to the possibilities and limitations of the different forms of representation of each category of functions, such as explanation of the role of parameters.

Development of functional thinking based on literature



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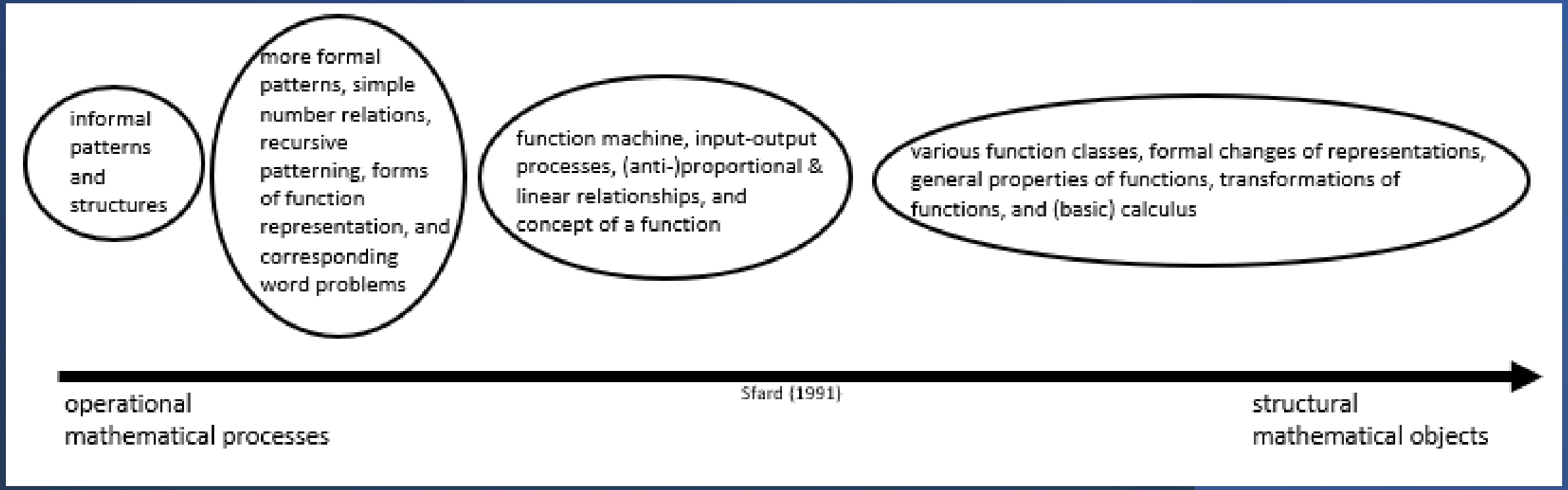

informal patterns and structures
more formal patterns, simple number relations, recursive patterning, forms of function representation, and corresponding word problems
function machine, input-output processes, (anti-)proportional & linear relationships, and concept of a function
various function classes, formal changes of representations, general properties of functions, transformations of functions, and (basic) calculus
Sfard (1991)
operational mathematical processes
structural mathematical objects

Common Features

Concepts: function, quantities, variables, functions in realistic contexts

Processes: Generalize, describe, reason, test, represent, analyze

Modes of representation: words, symbols, tables, graphs, diagrams

Functional Thinking *AND* Algebraic Thinking

Functional thinking is one of the five content areas of Kaput's (2005) model of early algebra and a core dimension of Kieran's (2017) model.

Functional thinking component of algebraic thinking

Vs

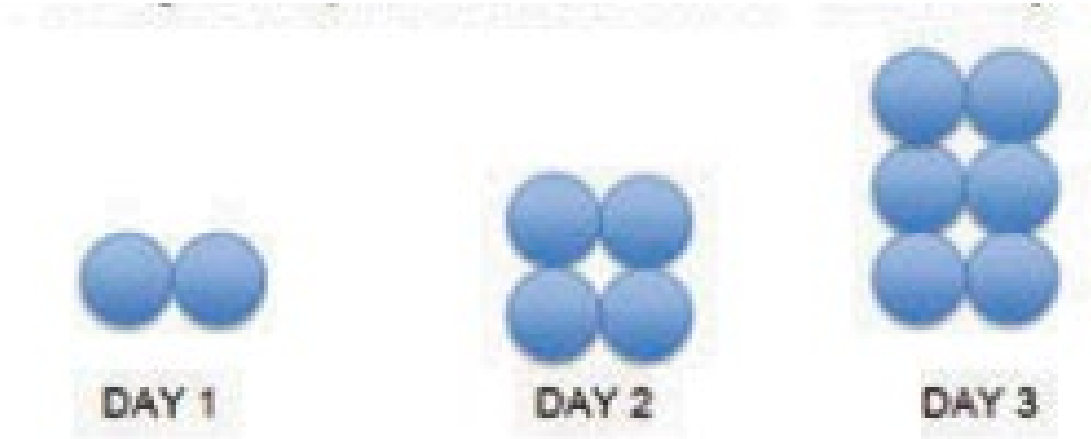
Functional thinking broader ability to which algebraic thinking contributes

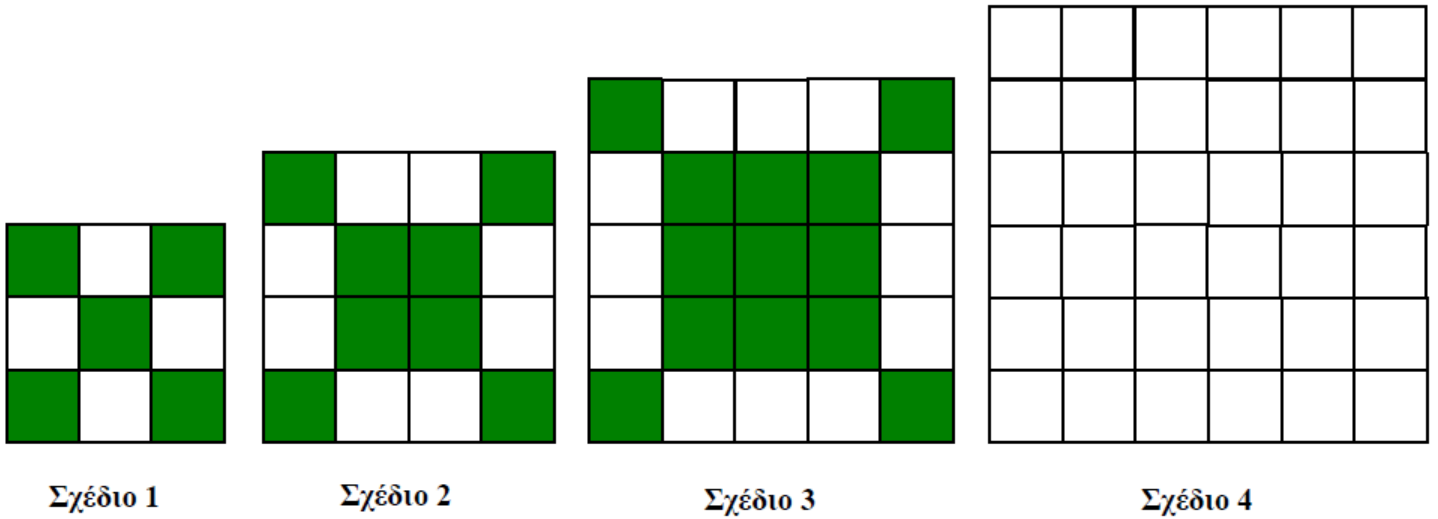
The goal of functional thinking is to deepen students' conceptual understanding and develop the ability to generalize

Ways of approaching problems involving functional relationships (Confrey & Smith, 1995)

- **Recursive reasoning:** each subsequent term in relation to the previous one (focus on the dependent variable). Search for regularities (Kieran, 2018).
- **Matching:** focusing on the relationship between the elements of the two sets and finding a rule that connects each value of the independent variable to the corresponding value of the dependent variable, forming a correspondence between the variable x and the variable y (Smith, 2008). It also includes the use of the rule to calculate remote terms, as well as to calculate the value of one variable, given the corresponding value of the other variable.
- **Covariation:** how quantities change simultaneously, coordinating the change of one quantity with the change of another, and understanding that the rule of covariation is the same for any values (Smith, 2008; Thompson & Carlson, 2017).

MACHINE D	
INPUT	OUTPUT
2	5
3	7
5	11
7	
10	





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Sample Answers

MACHINE D	
INPUT	OUTPUT
2	5
3	7
5	11
7	13
10	15

MACHINE D	
INPUT	OUTPUT
2	5
3	7
5	11
7	
10	

(a) Complete the table.

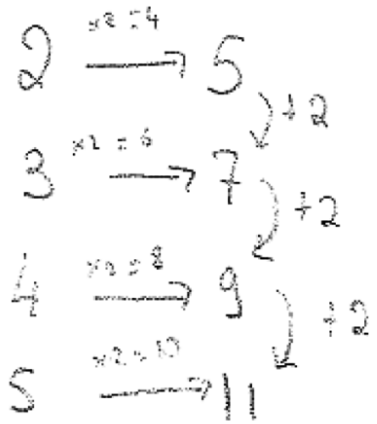
(b) Describe the machine's rule.

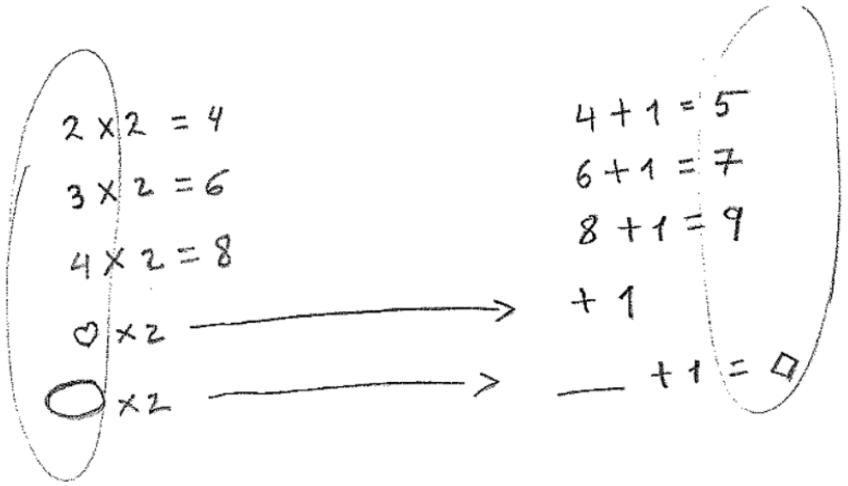
(c) Find the output value if the symbol of a heart was entered.

(d) Find the input value, if the symbol of a diamond was presented at the exit.

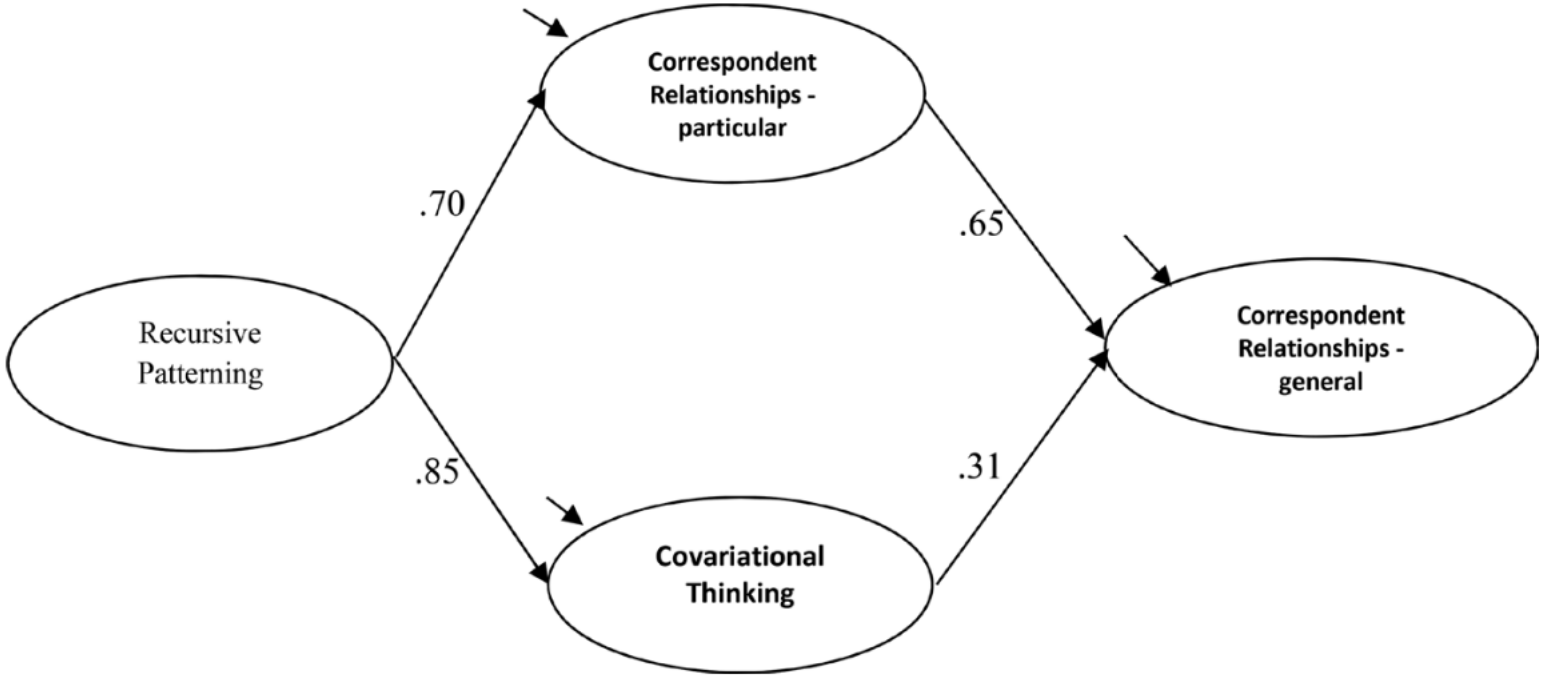
MACHINE D	
INPUT	OUTPUT
2	5
3	7
5	11
7	
10	

5×2=10
10+1=11





Relationships between approaches



Write questions for the pattern below. What approach does each question involve?

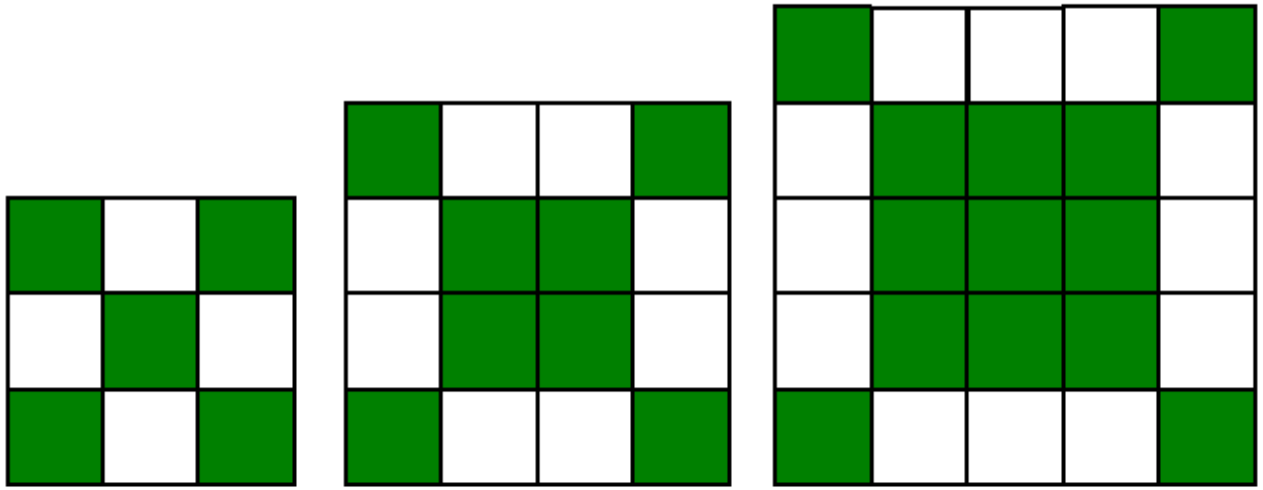
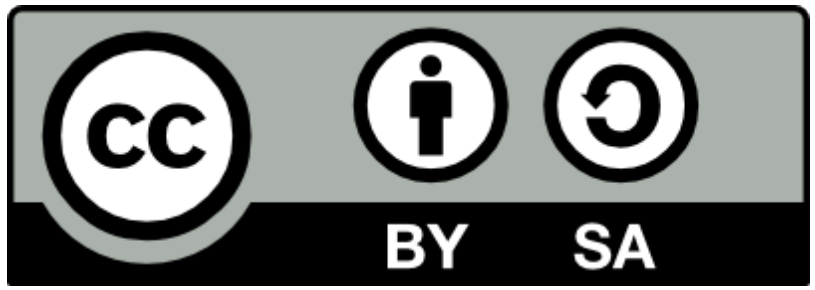


Figure 1 Figure 2 Figure 3



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Teacher Course Meeting 4 Patterns

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https://sites.google.com/view/seminarcy/home

Pattern

It is a predicted regularity involving a set of characteristics or relationships that remain constant within groups of numbers, shapes, sizes, or other mathematical and non-mathematical situations.

The relation “add 1 to a set (N)” will always result in N+1 regardless of whether the set is teddy bears, squares or numbers.

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Patterns Category

- Structure (form, rule)
  - ABAB..., ABCABC..., 2,4,6,2,4,6,...
- Material
  - Real, virtual, symbolic
  - Verbal
  - Audio
- Content
  - Color, shape, size
- Development
  - Growing and Repeating
  - Linear and circular

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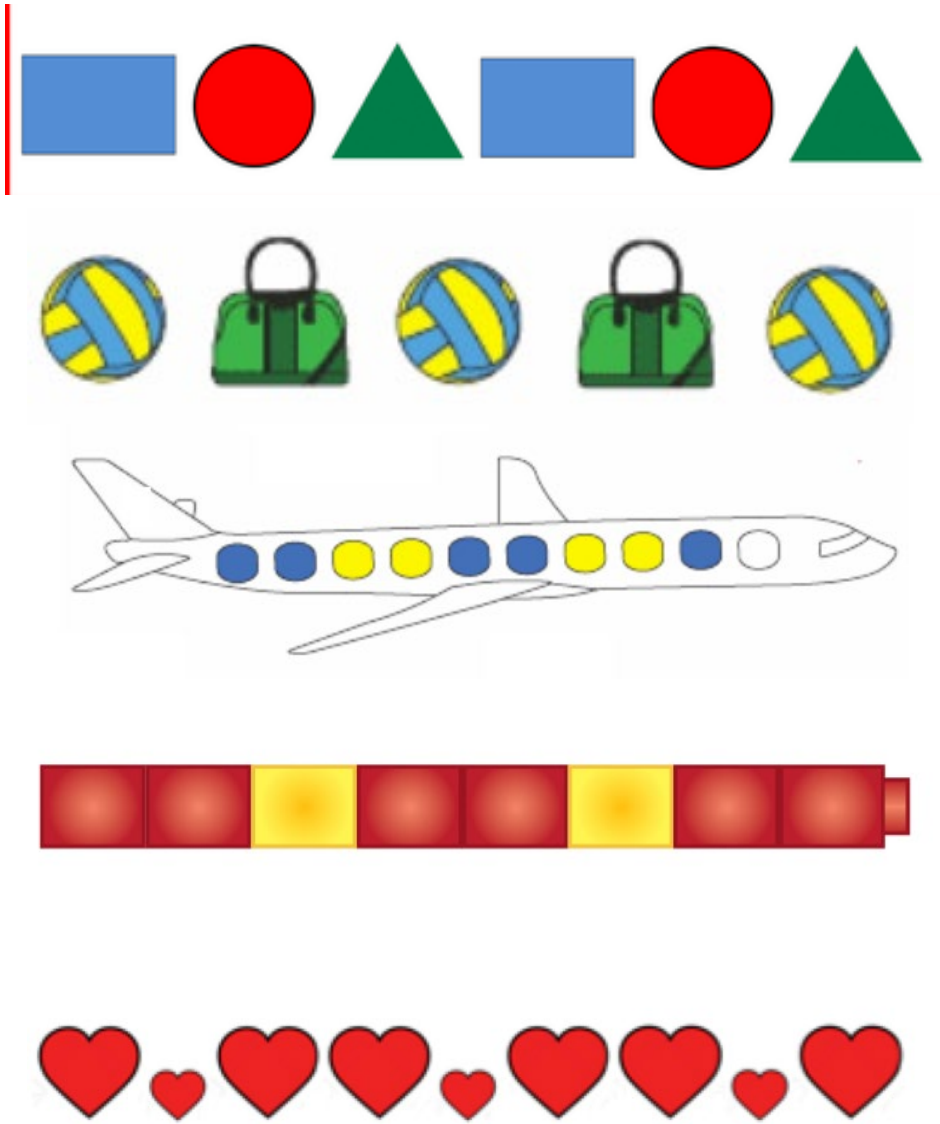
Patterns

- Repeating Patterns: Discrete unit of repetition
  - With shapes: circle, square, circle, square, ...
  - With colors: red, blue, blue, red, blue, blue, ...
  - With sounds: do, re, mi, do, re, mi, ...
  - With different sizes



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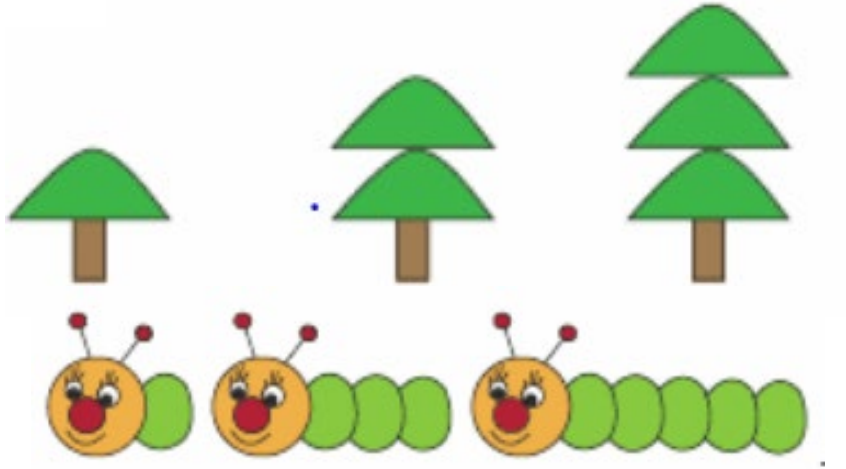
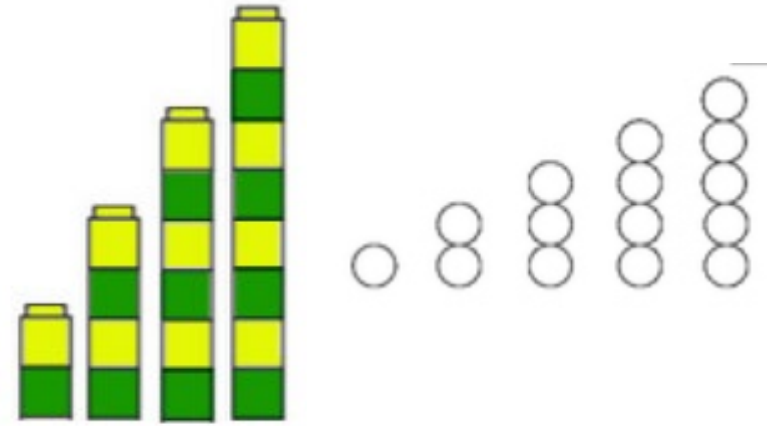
Patterns

- Growing Patterns: Increase or decrease in a systematic way
  - 2, 4, 6, 8, ...





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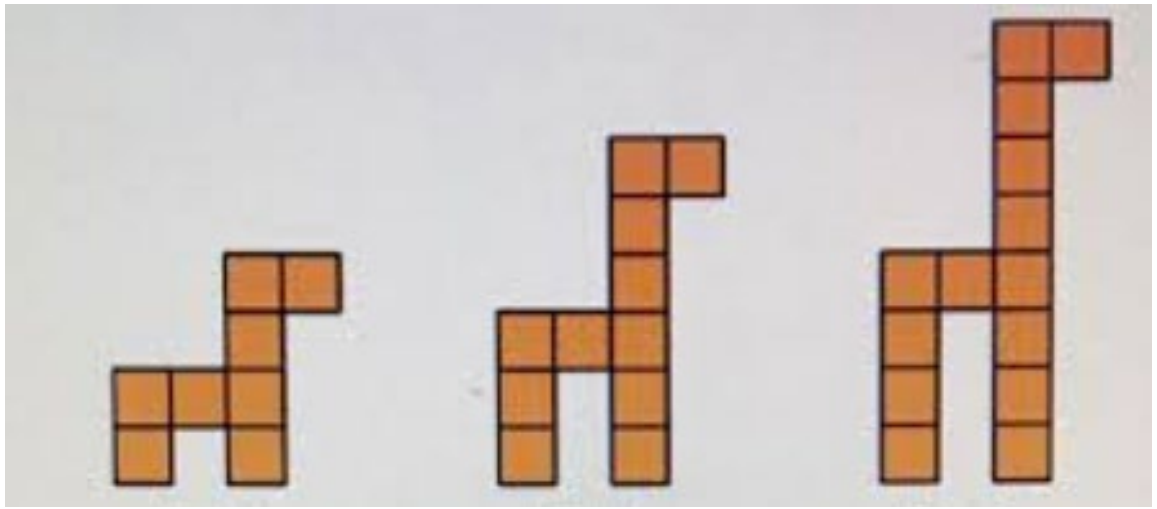

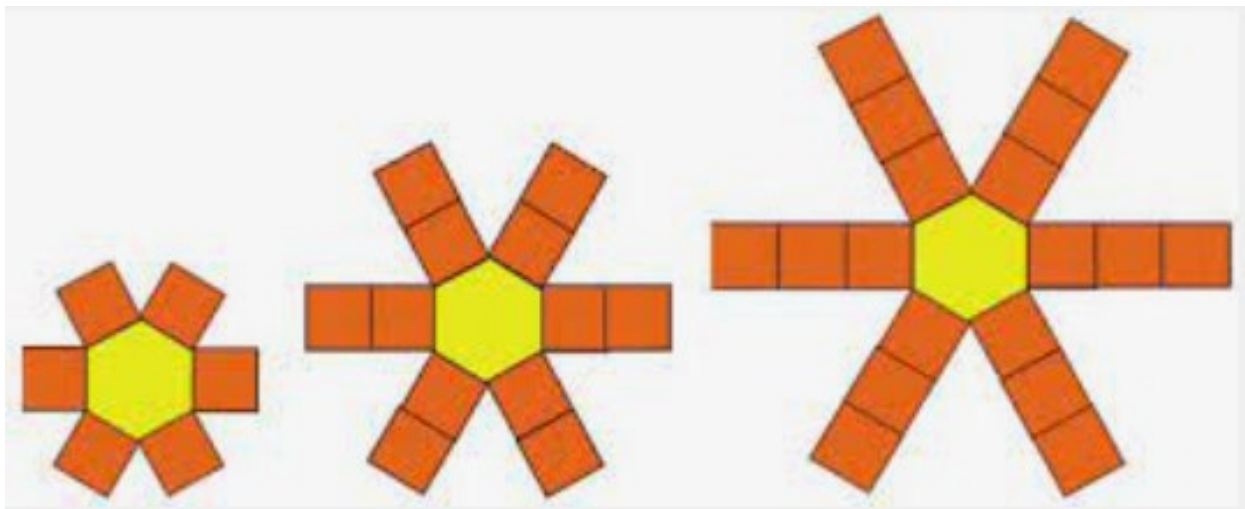

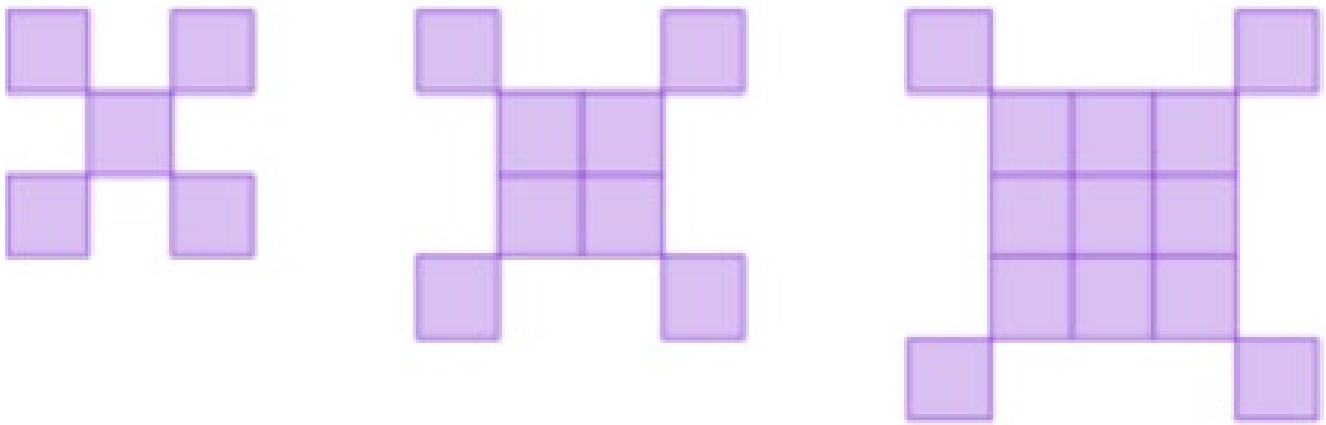

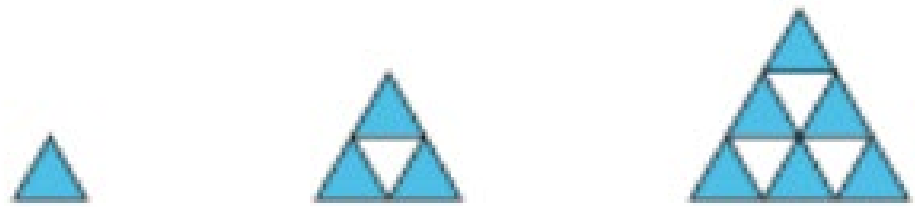

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Activities for …

- Observation of regularity
- Pattern recognition and description
- Pattern extension
- Pattern completion
- Pattern correction
- Pattern translation
- Pattern making
- Pattern generalization

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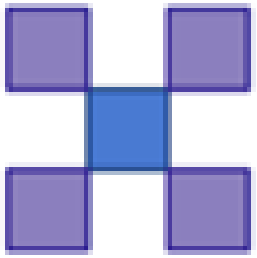

Figure 1

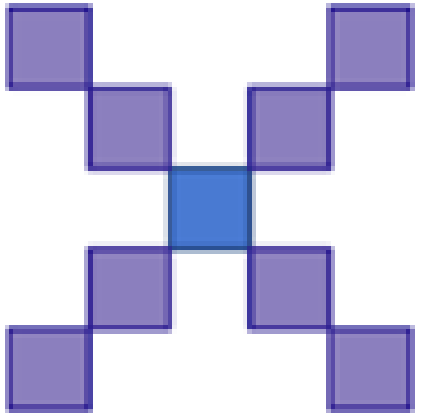

Figure 2

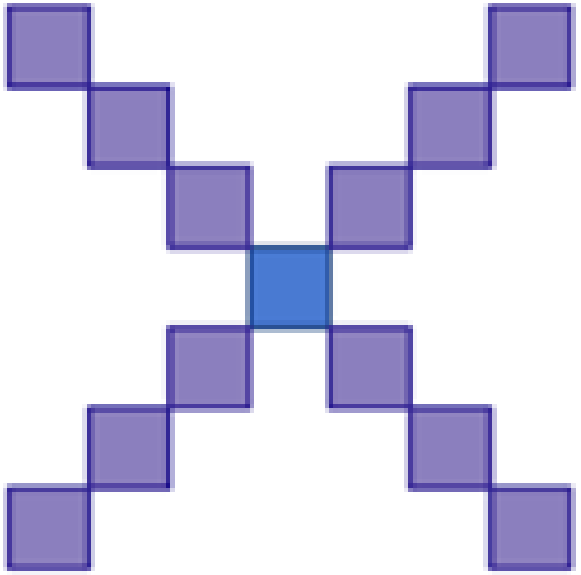

Figure 3

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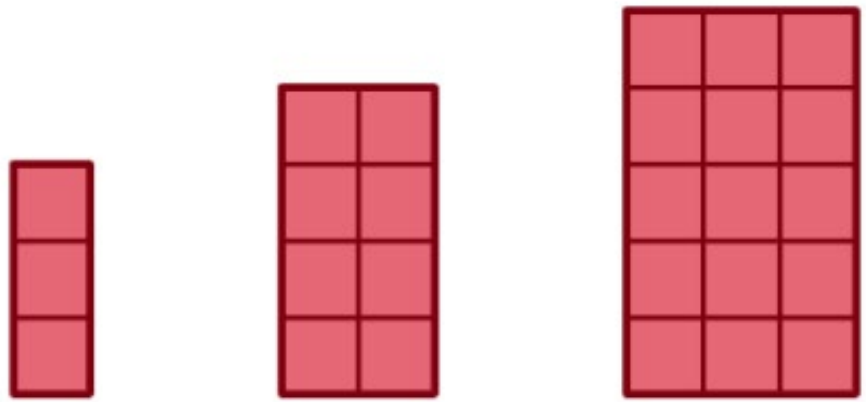

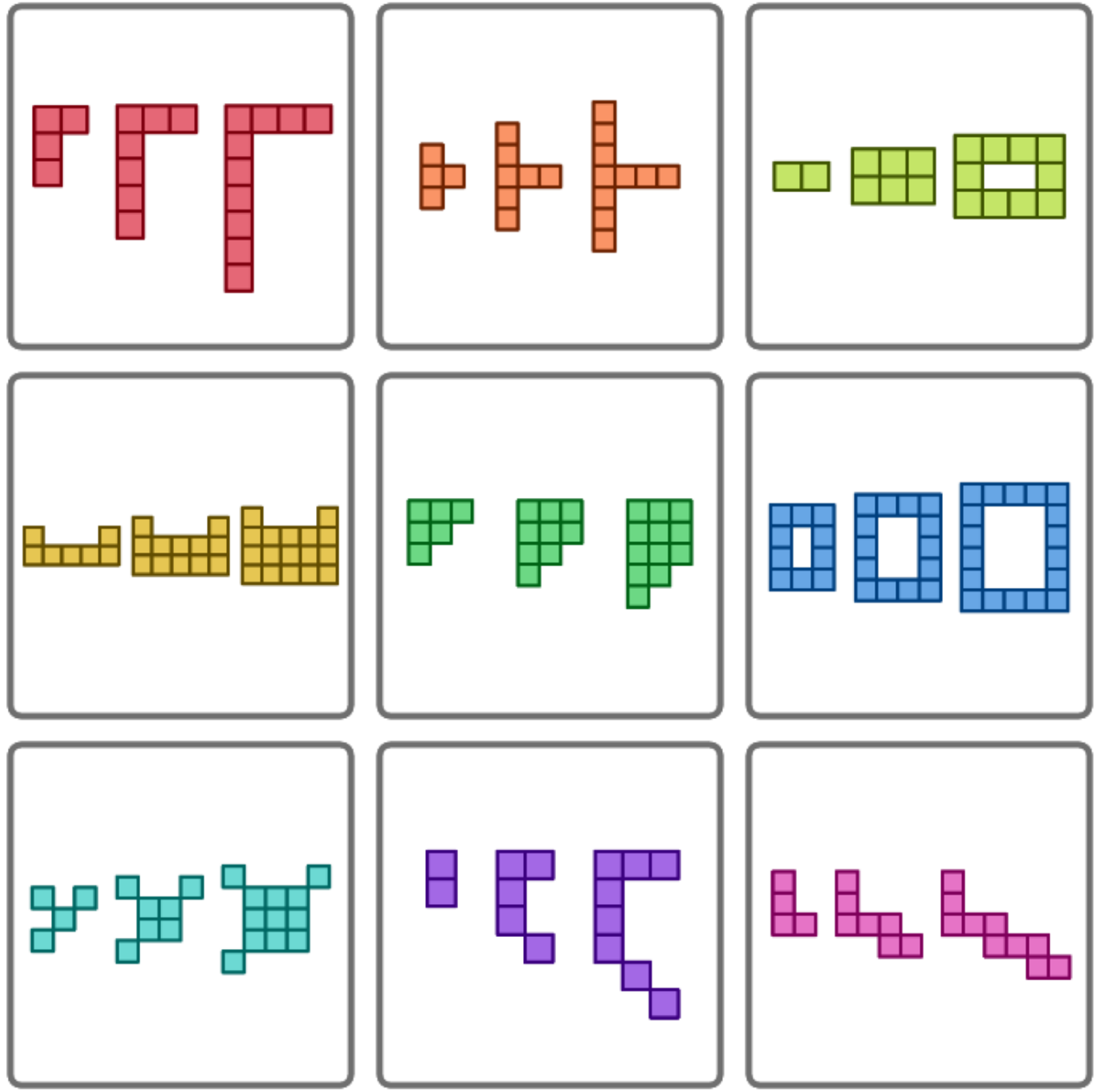

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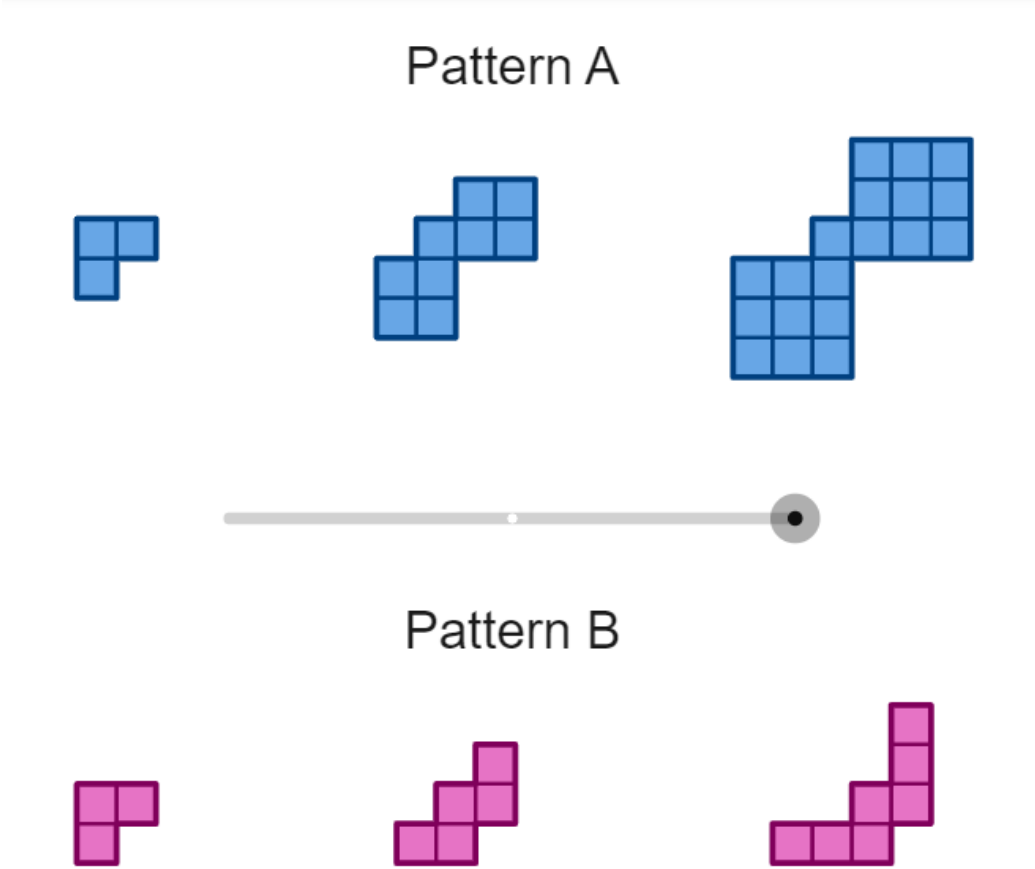

[Revisiting Visual Patterns, Part 2 • Activity Builder by Desmos]

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