

Teacher training course (6 h)

Workshop description

Some learning goals for mathematics transcend individual lessons and span many years of development. One such goal is to study relations between quantities: what does it mean for quantities to be dependent? In what ways can quantities be related? How do you express this using algebra and functions? Insight in these fundamentals is called functional thinking.



In this course we study how students can be supported in their development of functional thinking. There is particular attention for how this development stretches out from primary to upper secondary education. We discuss well-known and newly developed tasks that foster functional thinking. These tasks are based on the latest insight on the use of digital technology, inquiry, and movements (of the body) in mathematics education. Participants are invited to try out these tasks in their own classroom and reflect jointly on the ensuing outcomes and insights. As a result, the participants develop a general ability to foster functional thinking, and take home some interesting and useful new tasks, lessons, and classroom ideas for their students.

Target group

secondary school mathematics teacher. (Example for In-service Teacher course: 2 x 3h)

Possible sequence of the short teacher course:

Module 1 (3h):

Introduction to promoting functional thinking through situated learning environments

Key activities:

- study and discuss functional thinking
- study and give examples of the different aspects of functional thinking

Participants try out a module/ task in their classroom

Module 2:

(Observing) Functional thinking in classroom

Key activities:

- identify students' misconceptions and suggest ways to overcome them
- analyzing (common) difficulties and successes, strategies chosen by students

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module 1:

Introduction to promoting functional thinking through situated learning environments

Brief description In the introductory component of this two-part teacher course, participants will have the opportunity to become aware of their own conceptions about functional thinking, engage with the different aspects of functional thinking and the role of the function concept. Participants explore different learning environments.

Learning objectives Participants are able...

- to formulate in their own words what functional thinking entails.
- to identify the different aspects of functional thinking in learning environments
- to find opportunities for its development in (lower) secondary school (and in addition primary and upper secondary)
- to indicate how it is already addressed in the curriculum
- give examples that show the importance of functional thinking.

Structure of the module 1

- Phase 1: What is functional thinking? (Theory of functions)
- Phase 2: Recognizing theory in design
- Phase 3: Representations of functions
- Phase 4: How do we continue to work (in classroom)?

material PPT, tablet, beamer, internet connection

Example of possible time structure for a three-hour block (180 min) (other structures possible)

time	Phase/ Activity	Material/ media
Phase 1: Theory of functions		
5 minutes	Welcome, intro, agenda, learning goals	PTT: slide 1-2
10 min	Activity A - Introduction task: Engaging in an introduction task: Solving the tasks and answering the question: What functions are these? (Nomogram) → Freudenthal about function	videoclip of two hands PPT: slide 3-4
30 min	Participants are asked to answer two questions: What is functional thinking according to you? How it might relate to the function concept? <i>Input</i> for functional thinking (4 aspects & developing functional thinking & embodied design for function)	PPT: slides 5 - 15
Phase 2: Recognizing theory in design		
25-30 min	Activity B – Recognizing theory in design: Participants go through a digital environment (Hang WEIs nomograms) For each of the four aspects of function, they find and write down an item that is ideally situated to it. Participants discuss and exchange their findings.	Nomograms PTT: 16 - 17
10 min	<i>Input</i> on design principles for function & representation of functions	PPT: 18 - 21
Phase 3: Elaboration of the learning environments		
25-30 min	Activity C – Functions tasks: Participants find/ create one or more problems for undergraduate students that are solved using functional thinking, but in a different presentation from the one in which it is posed.	PTT: 22-24
15 min	Each group presents their problems	Whiteboard
5 min	Wrapping up, summary, general reflections on the tasks.	
Phase 4: How do we continue to work (in classroom)?		
30 min	Participants prepare for trying out and experiencing a module in their own classroom. For Preparation they watch some teaser videos.	PPT: 25-28
Distance phase/ Homework		
Each participant watches the rest of the teaser videos at home and chooses a module or at least one task from a module. The teaser videos can be found on the project website www.funthink.eu . It is included on the subpage of each learning environment		
Each participant tries out the module/ task in their classroom and gathers some feedback from their students and writes some personal reflections. (Optional: participant lets a colleague observe or uses video observation or/ and gathers students' documents)		

The participant then prepares a short PowerPoint presentation about their experiences, using the template. The presentation is sent in advance to the instructors

module 2: **Observing functional thinking in classroom**

Brief description	In the second component of this two-part teacher course, participants will have the opportunity to use student products and lessons recordings to deepen knowledge of functional thinking.
Learning objectives	Participants are able to... <ul style="list-style-type: none">• link at least one design criterion with teaching experiences.• analyze (common) misconceptions, difficulties and successes as well as chosen strategies• recognize differences between students in the learning process of functional thinking
Structure of the module 2	<ul style="list-style-type: none">• Phase 1: Exchange of teaching experience• Phase 2: Elaboration of the design principles• Phase 3: Strategies and difficulties of students working with LEs
material	PPT, tablet, beamer, internet connection

Example of possible time structure for a three-hour block (180 min) (other structures possible)

time	Phase/ Activity	Material/ media
Phase 1: Exchange of teaching experience (45-60 min)		
5 min	Welcome, goals, agenda	PPT 1-2
45-60 min	<p>Activity A - Exchange of experience. The instructor chooses 3 or 4 participants with different experiences to present their presentation for 3-5 minutes. Each presentation is followed by a round of feedback and discussion.</p> <p>Next, optionally, participants who tried the same module share experiences in small groups.</p> <p>The goal is to find out what was learned from the experiences in particular with respect to addressing functional thinking in classroom.</p>	<p>PPT 3-4</p> <p>Use the PPT “Report_classroom_experience_with_FunThink_module-1”</p>
Phase 2: Elaboration of the design principles		
15 min	<i>Input</i> for design principles. The instructor presents research findings regarding design principles based on the vision document. The instructor focuses at least on one design criteria (digital tools), but also mentions the other three.	PPT 5-9
20 min	<p>Activity B – Analyze a task:</p> <p>Analyze a task (maybe the one tried out in classroom) with respect to the design criteria. What is the added value of the tool? Does tool help to develop functional thinking? In what way? Could that be improved? Or when focusing on inquiry: What is the added value of the element of inquiry? What are the possible student strategies in the inquiry episode? How should the teacher anticipate on and deal with the episode of inquiry?</p> <ul style="list-style-type: none"> - One or two tasks to analyzed in various groups. Each half of the groups does one task - In the end: the instructor summarizes the findings in classroom conversation, and the findings are compared. 	<p>Tasks (tried out in classroom or other)</p> <p>PTT 10</p>
Phase 3: Strategies and difficulties of students working with LEs		
15 min	<p>Activity C – Watching a video from classroom learning episode: Participants watch a video and answer two questions: What kind of strategies do students use? What difficulties do they seem to face? In a classroom discussion the findings are shared and discussed.</p>	Video 11-13
10 min	Input about some known difficulties, students’ understandings of functional relationships (illustrated by the task from Activity C) and misconception from literature.	PPT 14-21

20 min Optional: Participants watch and discuss some more videos on students' misconceptions (from the classroom observation they made)

Phase 4: (Re)design your own activity, and wrapping up the course

35 min **Activity D – Designing activities:** PPT 22-23

Participants are asked to (re)design an activity that integrates the design principles of the project and different aspects of function, e.g. based on a task they will use in their teaching in the upcoming weeks. Depending on the time student just sketch the idea or work a bit more on the details. The participants and instructor have ad hoc discussions about the designs.

Finally, in a classroom discussion the instructor and participants share the main things they learned from the course. PPT 24-26
