Report on implementation phase in Cyprus

Module:	Function Machines Double Number-Line Distance-Time Patterns Qualitative interpretation of Graphs
Responsible Partner:	University of Cyprus
Grade Level/Age Range:	Grade 5 & 6
Sample size:	104
Brief Description of Testing / Intervention:	Intervention consisted of four 80-minute lessons in Grade 5 and five 80-minute lessons in Grade 6. Teaching in Grade 5 included the following modules: Function machines, Double-Number Lines, Distance-Time and Patterns. Teaching in Grade 6 included the following modules: Function machines, Double-Number Lines, Distance-Time, Patterns and Qualitative Interpretation of Graphs. **Learning Goals** Lesson 1 (Grades 5 and 6): Function machines The module engages students with the relationship between function machines, exploring the graphical representation and explaining the rule that follows the input and output values. Students conceptualize arithmetic operations as functions in an implicit way, functions as an input-output process, notice, generalize and express additive, multiplicative and linear relations and use functional expressions to model real-life scenarios. Lesson 2 (Grades 5 and 6): Double-number lines The module engages students with double number lines. Students engage in identifying and representing the rule of every number line, present it either with words, either with symbols and use them to find relation between the numbers in left and right axis. Then, they explore how dragging a point on a double number line creates a graph on a coordinate grid that represents the functional relation of the double-number line mapping. Lesson 3 (Grades 5 and 6): Distance-Time scenarios The module engages students with the relationship between distance and time, exploring the graphical representation as well as the rate of change in distance/time scenarios. Students conceptualize intuitively the formal definition of function in the context of distance-time scenario, interpret the graphical representation of distance-time scenarios, identify the

	relationship between distance-time and express it (verbally/symbolically) and create graphs of distance-time. Lesson 4 (Grades 5 and 6): Patterns The module engages students with growing patterns. Students engage in identifying and representing growing patterns, in finding recursive and functional relations. Students identify growing and repeating pattern, represent, and describe growing patterns using words, table, graph, extend growing patterns using different modes of thinking, identify covariation and correspondence relations in growing patterns, express the relations (verbally/symbolically) and generalize. Lesson 5 (Grade 6): Qualitative interpretation of graphs Students use graphs to model real-life scenarios, interpret intuitively how changing a graph modifies the quantities involved in a real-life scenario and represent linear relations. Students pose problems based on graphs that represent linear relations.

Method:

The implementation phase of the project in Cyprus involved 104 students (Grade 5, 50 students and Grade 6, 54 students; 49 girls and 55 boys) from the 7th Primary School in Lakatamia, Nicosia. The implementation phase took place in the period February-June 2023 and included the following: Administration of pre-test, teaching, administration of post-test and motivational test. All the lessons were delivered by members of the research team in collaboration with the mathematics teachers of the classes involved.

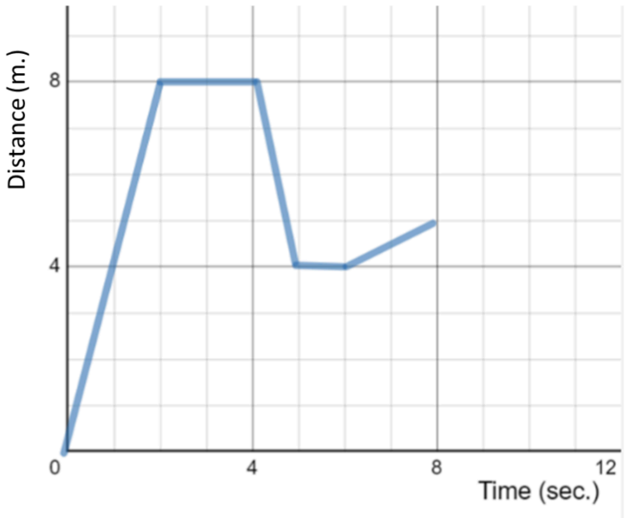
Pre-Test

It consisted of three items. The score of each item and the total score was weighed to 1 to enable direct comparisons.

Item 1 (see Figure 1) was based on an item used by Duijzer (2020) and measured graph interpretation and construction. Students observe a graph with data about a travelling car (distance-time). The first question required global and local interpretation of the graph as students had to identify which parts of the graph represented moving away or moving towards a person. The second question asked to identify when the car moves the fastest. The third question asked students to extend the graph for the following seconds based on a given description.

Figure 1

Ann plays with a remote-control car toy. The following graph presents the distance of the car from Ann in respect to time.



a. When was the car moving away from Ann and when towards Ann? Please explain.

b. When did the car move the fastest? Please explain.

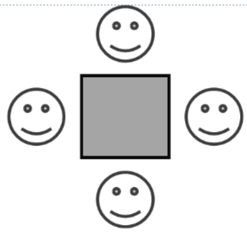
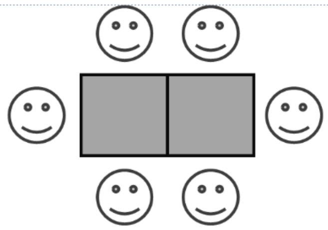
c. Complete the graph for the next four seconds based on the following:

"The car moved away from Ann for another one second and then moved towards her, without reaching her."

Item 2 (see Figure 2) was based on the Birthday Party item that was used by Blanton et al. (2015). The first question of the item required to find a term of a pattern that could be calculated based on a recursive, covariation or correspondence rule. The second one required calculating a far-transfer item and the third one asked student to provide the general rule of the pattern. A mean score was calculated for the first two questions.

Figure 2

Brady is having his friends over for a birthday party. He wants to make sure he has a seat for everyone. He has square tables.

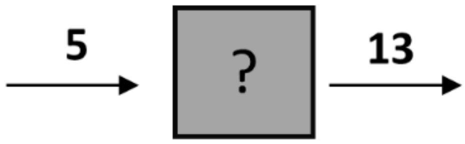
He can seat 4 people at one square table in the following way:		If he joins another square table to the first one, he can seat 6 people:
		

If Brady has 8 tables, how many people can he seat at his birthday party? And how about 20 tables? Can you find a rule that describes the relationship between the number of tables and the number of people who can sit at the tables?

Item 3 (see Figure 3) was developed for the purpose of the study based on ideas suggested by Pittalis et al. (2020) and Ng (2018) and measured students' ability to identify the numerical relation between two sets of values to find the input or output value of a function machine. Students were also asked to express the rule of the machine using symbols.

Figure 3

a. Find below a function machine. A number is entered, and the machine gives an output value based on a secret rule.



The table shows some inputs and outputs of this machine. Complete the empty cells. Show your calculations in the last column.

INPUT	OUTPUT	CALCULATIONS
0	3	
5	13	
7	17	
10	23	
12		
15		
	11	
	43	

b. John entered the symbol * in the machine. What will be the output? Please explain.

Post-Test

It included the three items of the pre-test and four additional ones, that referred directly to the intervention.

Motivational-Test

A motivational test was administered to students after the completion of the intervention. The test measured students' attitude to mathematics, their attitude to functional thinking, their mathematics self-esteem and functional thinking self-esteem.

Results and Discussion:

Results

A paired t-test showed that there was a statistically significant improvement between students' performance in the pre- and post-test. Table 1 shows means and standard deviations for the three items separately as well as cumulated (Total Score). Accordingly, Grade 5 students total score was .29 in the pre-test and .57 in the post-test, respectively. Grade 6 students' total score was .38 in the pre-test and .59 in the post-test.

Table 1

	Pre-Test		Post-Test		Significance
	Mean	SD	Mean	SD	
Total Score					
Grade 5	.29	.22	.57	.28	<.001
Grade 6	.38	.24	.59	.28	<.001
Item 1					
Grade 5	.16	.19	.42	.28	<.001
Grade 6	.18	.27	.44	.31	<.001
Item 2					
Grade 5	.40	.37	.65	.37	<.001
Grade 6	.51	.36	.69	.38	<.01
Item 3					
Grade 5	.30	.40	.65	.42	<.001
Grade 6	.48	.27	.70	.31	<.001

Grade 5 students' mean score in Item 1 improved from .16 to .42, while the improvement for Grade 6 students was from .18 to .44. In Item 2, Grade 5 students' mean score improved from .40 to .65 and Grade 6 from .51 to .69, respectively. In Item 3, Grade 5 students' mean score was .30 in the pre-test and .65 in the post-test, and .48 and .70 for Grade 6 students, respectively.

In the following section, we provide details for each sub-component of the items.

Item 1 – Function as covariation

Table 2 presents the means and standard deviations of the two groups of students in the three sub-components of Item 1. There was a statistically significant difference between the pre- and post-test results in all subcomponents of item 1 for both groups.

Grade 5 mean score in global-local interpretation questions was .18 in the pre-test and .42 in the post-test. The respective values for Grade 6 were .17 and .47. Grade 5 students' score in finding the fastest part of the trip was .19 in the pre-test and .43 in the post-test, while Grade 6 students mean scores were .20 and .30, respectively.

In respect to graph construction, Grade 5 students mean score was .09 in the pre-test and raised up to .45 in the post-test. Grade 6 students mean score was .18 in the pre-test and ended up to .50 in the post-test.

Table 2

	Pre-Test		Post-Test		Significance
	Mean	SD	Mean	SD	
Global-Local Interpretation					
Grade 5	.18	.26	.42	.34	<.001
Grade 6	.17	.31	.47	.37	<.001
Finding the fastest part of the trip					
Grade 5	.19	.25	.43	.26	<.001
Grade 6	.20	.28	.30	.27	<.05
Graph Construction					
Grade 5	.09	.26	.45	.43	<.001
Grade 6	.18	.36	.50	.43	<.001

Item 2 – Function as correspondence

Table 3 presents the means and standard deviations of the two groups of students in the two sub-components of Item 2. There was a statistically significant difference between the pre- and post-test results in extending patterns for Grade 5 students and finding the functional rule for both groups.

Table 3

	Pre-Test		Post-Test		Significance
	Mean	SD	Mean	SD	
Extending patterns					
Grade 5	.55	.48	.77	.39	<.001
Grade 6	.68	.42	.76	.44	
Finding the general rule					
Grade 5	.25	.40	.53	.44	<.001
Grade 6	.33	.44	.63	.46	<.001

In grade 5, the mean score in extending patterns was .55 in the pre-test and .77 in the post-test. The respective values for Grade 6 were .68 and .76 (not statistically significant difference). In respect to finding and expressing with symbols the general rule of the machine, Grade 5 students mean score was .25 in the pre-test and raised up to .53 in the post-test. Grade 6 students mean score was .33 in the pre-test and .63 in the post-test.

Analysis showed that there was a significant change in the type of strategies adopted by students between the pre- and post-test to extend the pattern and express the general rule (Chi-Square = 33.59, df = 9, $p < .001$). Table 4 presents the frequency of the strategies used by students in the pre- and post-test. For instance, in the pre-test 57.7% of the students used a recursive strategy and only 19.2% of the students used a correspondence strategy. In the post-test, 33.7% adopted a recursive strategy and 39.4% preferred a correspondence one.

Table 4

Description of Strategy	Pre-test		Post-test	
	Absolute Frequency	Relative Frequency	Absolute Frequency	Relative Frequency
Recursive	60	57.7%	35	33.7%
Covariational	6	5.8%	15	14.4%
Correspondence	20	19.2%	41	39.4%
Missing	18	17.3%	13	12.5%

Item 3 – Function as input-output

Table 5 presents the means and standard deviations of the two groups of students in the three sub-components of Item 3. There was a statistically significant difference between the pre- and post-test results in all subcomponents of item 3 for both groups.

Table 5

	Pre-Test		Post-Test		Significance
	Mean	SD	Mean	SD	
Finding the output value					
Grade 5	.37	.47	.71	.43	<.001
Grade 6	.49	.49	.71	.45	<.01
Finding the input value					
Grade 5	.33	.46	.64	.45	<.001
Grade 6	.51	.48	.70	.43	<.01
Rule					
Grade 5	.20	.38	.61	.49	<.001
Grade 6	.39	.49	.61	.49	<.001

Grade 5 mean score in finding the output-value of the function machine was .37 in the pre-test and .71 in the post-test. The respective values for Grade 6 were .49 and .71. In finding the input value for given output values, Grade 5 students pre- and post-test scores were .33 and .64, and .51 and .70 for Grade 6, respectively.

Regarding the type of strategies adopted by students between the pre- and post-test, analysis shows that there was a significant change (Chi-Square = 130.66, df = 88; p<.01). Table 6 presents the frequency of the strategies used by students in the pre- and post-test to find the number of persons for 20 tables. For instance, in the pre-test only 28% of the students used a correspondence general strategy (with or without correct manipulation of symbols), while in the post-test the respective percentage was 21%.

Table 6

	Description of Strategy	Pre-test		Post-test	
		Absolute Frequency	Relative Frequency	Absolute Frequency	Relative Frequency
1	Fully Correct				
	Correspondence general	20	19.2%	53	51%
	Correspondence general with wrong manipulation of symbols	9	8.7%	10	9.6%
2	Partially Correct				
	Correspondence particular	3	2.9%	1	1%
	Expressing the rule based on covariation	1	1%		
3	Wrong				
	Variation of only one	1	1%		
	quantity	4	3.8%		
	Repeat the pattern of the				
	given values	18	17.3%	13	12.5%
	Identification of different				
	additive rules	11	10.6%	7	6.7%
	Random calculations	3	2.9%	3	2.9%
	Correct completion of table				
	without showing	4	3.8%	1	1%
	calculations				
	Other wrong strategy				
4	Missing (students did not complete the task)	30	28.8%	16	15.3%

Motivational Factors

Analysis showed that students had a positive attitude towards mathematics and functional thinking lessons. Sat the time of the post-test, the mean value of their attitude towards mathematics and functional thinking were 3.45 (out of 5) (SD=1.05) and 3.33 (SD=1.19), respectively.

Further, the present study showed that mathematics self-esteem was high (M = 3.83, SD=.89). The mean value of students' self-esteem in functional thinking concepts was significantly lower (Mean = 3.50, SD=.78).

Effectiveness of the Intervention

We conducted a multiple full factorial analysis of variance to examine whether the program was effective for all students independently of their initial functional thinking level, grade, and sex. To do so, we used as a dependent variable each student's total gain (difference between pre and post-tests overall score). Analysis showed that there was no difference in students' gain due to their initial functional thinking level, grade or sex.

Further, we conducted a regression analysis to examine the effect of student's attitude towards mathematics and functional thinking and student's self-esteem in mathematics and functional thinking. Analysis showed that none of these factors was a predictor of their gain score.

Discussion

The results of this study provide compelling evidence that the intervention program in Cyprus contributed to developing students' functional thinking. There was a statistically significant difference in students' overall performance between the pre- and the post-test for Grade 5 and Grade 6 students. The difference between pre- and post-test results was statistically significant for all three items indicating that the program contributed to further developing students' understanding of function as an input-output, covariation and correspondence. It should be noted that Grade 5 students mean score was lower than Grade 6 students, but both groups had almost the same performance in the post-test, indicating that even Grade 5 students functional thinking can be developed. Further, the effectiveness of the intervention for each student was not related to the initial functional thinking level, grade, sex, attitude towards mathematics and functional thinking and self-esteem in mathematics and functional thinking.

Besides the significant improvement of students' performance, analysis showed a significant change in the strategies adopted by students. For instance, in items 2 and 3 students in the pre-test used mainly recursive strategies to identify the relation between the involved variables, while in the post-test most students adopted more advanced ones, such as correspondence or covariation strategies.

In conclusion, our study has shown promising results in the use the adopted modules for fostering functional thinking. The evidence presented herein paves the way for further explorations in the field and the development of innovative, effective instructional designs in mathematical education.

References

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