Report on implementation phase in Slovakia

Module:	Coordinates, Change is change, Marbles, Various Vessels
Responsible Partner:	UPJS Kosice, Slovakia
Grade Level/Age Range:	Grade 9
Sample size:	23
Brief Description of Testing / Intervention:	Intervention consisted of eight 45-minute lessons in Grade 9. It included the following modules: Coordinates, Change is change, Marbles, Various vessels. Each module had 2 lessons. These 2 lessons were consecutive. There was usually a 1-week break between each module (due to various events taking place at the school). Lesson 1, 2: Coordinates The lessons provided the space to develop an idea of the coordinate system, which is necessary for further work with functions. It is a way for students to come into contact with the input-output aspect and its graphical representation - a point in the coordinate system. Lesson 3, 4: Change is change The lesson plan aimed at understanding the slope or at creating a good basis for understanding it in the upper grades. In terms of functional thinking, the dominant aspect is covariance, in which students focus on comparing the change on the x and y axes. The correspondence aspect is also supported. Lesson 5, 6: Marbles By modelling a real situation, students got a first idea of a linear function. This is an increasing linear function whose domain is non-negative integers. The focus was mainly on the covariation aspect of the concept of a function - the change in volume of a container as a function of the number of balls placed in it. Students worked with different representations of a function (table, graph, verbal description, possibly with a formula). Lesson 7, 8: Various Vessels In the lesson, students explored the relationship between the volume of water poured into a vessel and the height of the water in the vessel (the covariance aspect of the concept of function - the dependence of the height of the water in the vessel on the volume of water poured into the vessel). Students described this relationship for different vessel shapes. They used different representations of the concept of function (graph, table, verbal description).

Method:

The teaching experiment was carried out with students of a grade nine class (14-15 years old) from a school in a village next to Kosice. The starting point and preliminary knowledge of the students were: having experience with reading off numbers from number lines; working with tables, forming an addition or subtraction expression to represent the relationship between two quantities; using a ratio to describe the relationship between two quantities; writing algebraic expressions to describe a real-world situation; having partial experience with reading from a graph.

The implementation phase took place in the period April-May 2023 and included the following: pre-test, teaching, post-test, a questionnaire about what students liked and disliked while working in the classroom with the learning environments. There were 23 students who completed the pre- and post-test before and after the experiment. The teacher who led the lessons had been teaching for 5 years. She collaborated with the research team and participated in a whole-day multiplier event in February 2023, which focused on learning environments from the FunThink project.

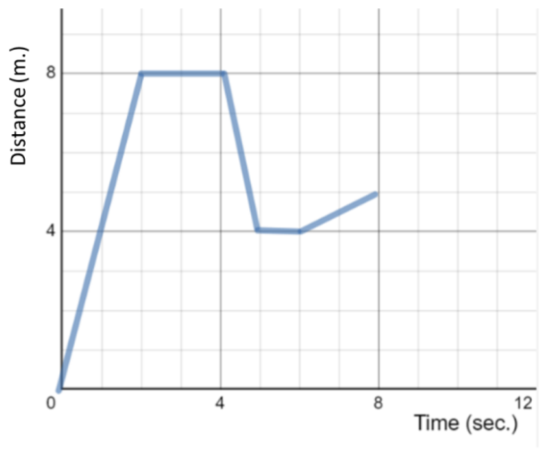
Testing

Pre-test and post-test consisted of three items.

Item 1 (see Figure 1) was based on an item used by Duijzer (2020) and measured graph interpretation and construction. Students observe a graph with data about a travelling car (distance-time). The first question required global and local interpretation of the graph as students had to identify which parts of the graph represented moving away or moving towards a person. The second question asked to identify when the car moves the fastest. The third question asked students to extend the graph for the following seconds based on a given description.

Figure 1

Ann plays with a remote-control car toy. The following graph presents the distance of the car from Ann in respect to time.



a. When was the car moving away from Ann and when towards Ann? Please explain.

b. When did the car move the fastest? Please explain.

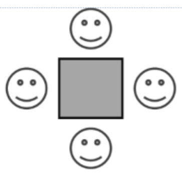
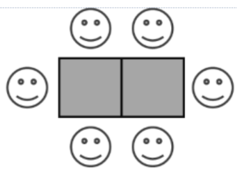
c. Complete the graph for the next four seconds based on the following:

"The car moved away from Ann for another one second and then moved towards her, without reaching her."

Item 2 (see Figure 2) was based on the Birthday Party item that was used by Blanton et al. (2015). The first question of the item required to find a term of a pattern that could be calculated based on a recursive, covariation or correspondence rule. The second one required calculating a far-transfer item and the third one asked student to provide the general rule of the pattern.

Figure 2

Brady is having his friends over for a birthday party. He wants to make sure he has a seat for everyone. He has square tables.

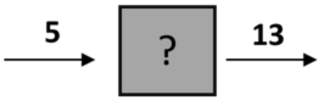
He can seat 4 people at one square table in the following way:		If he joins another square table to the first one, he can seat 6 people:
		

If Brady has 8 tables, how many people can he seat at his birthday party? And how about 20 tables? Can you find a rule that describes the relationship between the number of tables and the number of people who can sit at the tables?

Item 3 (see Figure 3) was developed for the purpose of the study based on ideas suggested by Pittalis et al. (2020) and Ng (2018) and measured students' ability to identify the numerical relation between two sets of values to find the input or output value of a function machine. Students were also asked to express the rule of the machine using symbols.

Figure 3

a. Find below a function machine. A number is entered, and the machine gives an output value based on a secret rule.



The table shows some inputs and outputs of this machine. Complete the empty cells. Show your calculations in the last column.

INPUT	OUTPUT	CALCULATIONS
0	3	
5	13	
7	17	
10	23	
12		
15		
	11	
	43	

b. John entered the symbol * in the machine. What will be the output? Please explain.

Results and Discussion:

Result

The results of our study indicate improvements in students' functional thinking as reflected in the pre- and post-test scores. Moreover, for item 3c the number of blank answers decrease. The increases in scoring proportions and decrease in blank answers from item 3c imply that the students were not only able to attempt the problems but also solve them correctly.

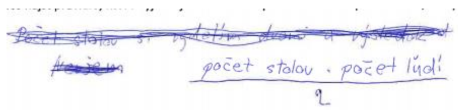
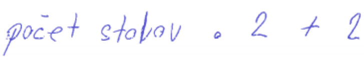
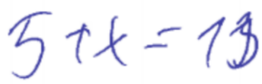
The following table shows the proportional increase in scores.

Item	Pre-test	Post-test
1a - correctness	21,7%	26,1%
1a - interval notation[1]	13,0%	26,1%
1b - correctness	60,9%	87,0%
1b - interval notation	13,0%	21,7%
1c - correctness	56,5%	65,2%

[1] Percentage of students who used the correct mathematical notation for intervals

2a - correct numbers	69,6%	78,3%
2b - rule correct	30,4%	52,2%
3a - correct numbers	65,2%	78,3%
3b - correct numbers	69,6%	73,9%
3c - correctness	60,9%	69,6%
3c - blank	30,4%	17,4%

A clear example of developed covariation and correspondence aspect of functional thinking can be seen in the students' responses to items 2b and 3c. In the pre-test, a student was not able to correctly find a general rule. However, in the post-test, the same student was able to correctly figure out the full rule. This shows a deepened understanding and improved ability in covariational reasoning, supporting our claim that the intervention was effective in improving students' functional reasoning.

Item	Pre-test	Post-test
2b	 If the number of tables is 8 then the number of people is 2 times more + 2	 2 · number of tables + 2
2b	 number of tables · number of people --- 2	 number of tables · 2 + 2
3c		

Discussion

The results of this study provide evidence that the proposed learning environments contribute to the development of students' functional thinking. The improvement in student performance on the post-test compared to the pre-test demonstrates the potential of the used environments to develop functional thinking.

The individual learning environments respected the design principles to foster functional thinking as described in the Vision document. From the students' responses to the questionnaire administered at the end of the intervention, we conclude that the success of the intervention is related to the phenomenologically rich situations and to the linking of the embodied experience of the digital-embodied tasks, which provide a concrete experience in combination with the abstract nature of functional thinking.

In addition to the improvement in students' performance, the analysis also showed a change in the strategies used by the students. For example, in items 2 and 3 of the pre-test, students focused mainly on filling in the values (usually using recursive strategies to determine the relationship between the variables involved), whereas in the post-test, more students used more advanced strategies such as correspondence or covariation.

In conclusion, our study has shown promising results in the use of the mentioned learning environments to support functional thinking. These results open the way for further research in this area and for the development of further innovative, effective instructional designs in mathematics education.