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Report on Teacher Training Course in Cyprus

Responsible Partner:	University of Cyprus
Participants	**Pre-service and in-service Course** 12 in-service and 8 pre-service teachers **Undergraduate students Course** 20 Year 4 students – Department of Education, University of Cyprus
Brief Description of Training Course:	The training course for **pre-service and in-service** teachers consisted of four 2-hour sessions. The course took place in the mathematics education lab. The training course for undergraduate **students** consisted of six 75-minutes sessions. The course was part of the mathematics specialization course EDU 473, Didactics of Arithmetic and Algebra. The courses included the following modules: **Module 1** Introduction – Aspects of functional thinking **Module 2** From variational to covariational reasoning: input-output, correspondence **Module 3** Covariation and correspondence in real-life scenarios, modeling distance-time stories **Module 4** Aspects of functional thinking in patterning situations

Method:

The participants were asked to fill in an online questionnaire (see below “online questionnaire") regarding functional thinking before the course’s first session. Participants were asked to answer the same questionnaire after the completion of the course. In addition, the participants were asked to evaluate the teacher training course and provide feedback (see below “feedback questionnaire").

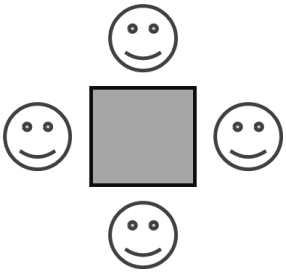
Online questionnaire

The questionnaire included the following questions:

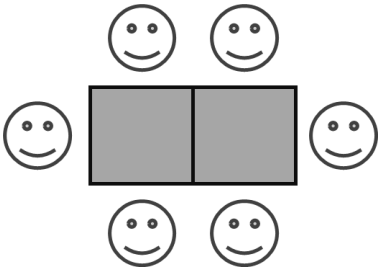
1. Have you ever encountered the term functional thinking? If so, what do you consider functional thinking to be? If not, what do you expect it to be? Expand your answer.
2. Name five words or expressions you think of when considering functional thinking.
3. From what age, in your opinion, is it possible to develop functional thinking as it was explained earlier? Justify your answer.
4. What topics in the mathematics curriculum (and in which grades) are relevant for functional thinking development? Justify your answer.
5. What do you consider to be a goal when teaching about functions? Expand your answer.
6. (a) Solve the following problem:

   Brady is having his friends over for a birthday party. He wants to make sure he has a seat for everyone. He has square tables.

   He can seat 4 people at one square table in the following way:

   If he joins another square table to the first one, he can seat 6 people:

   If Brady has 8 tables, how many people can he seat at his birthday party? And how about 20 tables? Explain your answer.

(b) Suggest three different questions related to the seating task above that could be posed to 5th grade students. Explain the purpose of each question.

Feedback Questionnaire

Please rate the following statements in context of the teacher training course. (1 – Absolutely Disagree / 5 – Absolutely Agree):

	1	2	3	4	5
1. I learned interesting things during the teacher training course about mathematics.	○	○	○	○	○
2. I learned interesting things during the teacher training course about teaching.	○	○	○	○	○
3. The knowledge I gained is useful for my professional development.	○	○	○	○	○
4. The teacher training course, as implemented, was appropriate and effective.	○	○	○	○	○
5. The knowledge I gained is useful for the teaching of Mathematics.	○	○	○	○	○
6. I will use the teaching material developed by the project in my teaching.	○	○	○	○	○
7. I will use the digital tools developed by the project in my teaching.	○	○	○	○	○
8. The digital tools of the project are interesting.	○	○	○	○	○
9. The digital tools of the project facilitate the development of conceptual understanding ofmathematics..	○	○	○	○	○
10. I would recommend this training courseto a colleague of mine.	○	○	○	○	○

Please answer the following questions.

1. What did you like most about this teacher training course?

2. What would you suggest changing in the structure of the teacher training course?

3. What other suggestions would you like to share with us regarding this teacher training course?

Results and Discussion:

Results

1. Have you ever encountered the term functional thinking? If so, what do you consider functional thinking to be? If not, what do you expect it to be? Expand your answer.

Table 1 presents the participants' responses before and after the completion of the training course. The participants mentioned various aspects of functional thinking in their answers. Coding was conducted based on a comprehensive coding framework that was developed by all participating universities.

Before the course, 50% of the pre- and in-service teachers mentioned that they never heard the term before and the other 50% mentioned one aspect of functional thinking (FT). Three of the teachers referred to FT as the correspondence relation between two quantities, three of them mentioned that FT is important in different domains of mathematics, two students explained that FT is an overarching idea and two of them provided an incomplete description of function. In respect to undergraduate students, 90% stated that they have never heard about FT before and only 5% mentioned more or less concrete ideas of FT.

After the completion of the course, most teachers and undergraduate students mentioned specific aspects of FT. In particular, 28% of the teachers referred to the FT as input-output, 22% referred to function as a dynamic process of covariation. Further, 17% stated that functions are involved in different domains of mathematics, 22% included the aspect of patterns in their description, 17% mentioned that FT involves different kinds of representations such as graphs and 17% referred to specific concepts that are related to functions, such as variables and equations.

In respect to undergraduate students, 12% emphasized the aspect of FT as input-output, 18% as dynamic process of covariation and 59% as a correspondence relation between two quantities/variables. In addition, 18% mentioned that functions are important in different domains of mathematics, 24% referred to patterns, 12% mentioned the role of representations and 35% talked about concepts that are related to functions.

Table 1

	Before the course		After the course	
Pre- and in-service teachers	Never heard before	10/20 (50%)	Never heard before	-
	Correspondence relation between two quantities	3/20 (15%)	Function as an input-output	5/18 (28%)
	It involves functions in different domains of mathematics	3/20 (10%)	Function as a dynamic process of covariation	4/18 (22%)
	Functional thinking as an overarching idea	2/20 (10%)	Correspondence relation between two quantities	5/18 (28%)

	Incomplete description of function	2/20 (10%)	It involves functions in different domains of mathematics	3/18 (17%)
			Patterns	4/18 (22%)
			Dealing with representations	3/18 (17%)
			Semantic and syntactic elements related to functions	3/18 (17%)
Undergraduate students	Never heard before	18/20 (90%)	Never heard before	-
	Correspondence relation between two quantities	1/20 (5%)	Function as an input-output	2/17 (12%)
	It involves functions in different domains of mathematics	1/20 (5%)	Function as a dynamic process of covariation	6/17 (18%)
			Correspondence relation between two quantities	10/17 (59%)
			It involves functions in different domains of mathematics	3/17 (18%)
			Patterns	4/17 (24%)
			Dealing with representations	2/17 (12%)
			Semantic and syntactic elements related to functions	6/17 (35%)

2. Name five words or expressions you think of when considering functional thinking.

Table 2 presents the words/expressions mentioned by teachers and undergraduate students in the online questionnaire before and after attending the teacher course. It should be noted that teachers and students before the teacher course referred mainly to the concept of function, algebra, reasoning, graphs, variables, and others. After the completion of the course, teachers and students referred to ideas that are related with specific aspects of function and types of activities. For instance, participants referred to function as input-output (5 out of 18

for teachers and 6 out of 17 for students), covariation (8 out of 18 for teachers and 14 out of 17 for students), and correspondence (8 out of 18 for teachers and 5 out of 17 for students). Participants also mentioned patterns (10 out of 18 for teachers and 5 out of 17 for students), relations (7 out of 18 for teachers and 7 out of 17 for students) and graphs (6 out of 18 for teachers and 5 out of 17 for students). Participants included in their description the concept of variables (5 out of 18 for teachers and 8 out of 17 for students).

Table 2

		Teachers		Students	
		Before (out of 20)	After (out of 18)	Before (out of 20)	After (out of 17)
1	Function	7	5	16	6
2	Algebra	4	-	1	-
3	Programming	1	-	-	-
4	Reasoning	1	-	4	-
5	Relations	3	7	3	7
6	Input/output	1	5	-	6
7	Covariation	1	8	-	14
8	Numbers	1	-	-	-
9	Operations/calculations	2	-	2	-
10	Sets	2	-	-	-
11	Generalization	1	-	-	7
12	Correspondence	3	8	-	5
13	Unknown quantities	1	-	1	-
14	Domain/range	2	-	-	-
15	Variables	4	5	5	8
16	Graph	1	6	5	5
17	Equation	1	-	1	-
18	Pattern	1	10	1	5
19	Proportions	-	-	1	4
20	Structure	-	-	-	6

3. From what age, in your opinion, is it possible to develop functional thinking as it was explained earlier? Justify your answer.

Table 3 presents participants' suggestions regarding the appropriate age to develop FT, before and after attending the teacher course. Before the course, most of the teachers suggested that FT can be developed from Grade 1 (6 out of 20), five of them from Grades 3-4, and the rest of them (7 out of 20) from Grade 5. After the course, a great majority (10 out of 18) suggested that FT can be developed from kindergarten and the rest of them from lower primary school. Before the course, most of the undergraduate students mentioned that FT can be developed from upper primary (13 out of 20), while after the course 7 out 17 stated that FT can be developed from kindergarten or Grade 1. Participants did not justify their answer. One teacher and two students, before the course, stated that FT can be developed from Grade 7.

Table 3

		Teachers		Students	
		Before (out of 20)	After (out of 18)	Before (out of 20)	After (out of 17)
1	Kindergarten	-	10	-	2
2	Grade 1	6	4	-	5
3	Grade 2	-	-	1	-
4	Grade 3	2	4	4	-
5	Grade 4	3	-	-	1
6	Grade 5	3	-	9	8
7	Grade 6	3	-	4	1
8	Grade 7	1	-	2	-
9	No answer	2	-	-	-

4. What topics in the mathematics curriculum (and in which grades) are relevant for functional thinking development? Justify your answer.

Table 4 presents the topics mentioned by teachers and students as relevant for the development of FT, before and after the course. In respect to teachers, before the course, most of them mentioned the domain of algebra (10 out of 20). After the course, besides algebra, teachers mentioned geometry (4 out of 18), statistics-probability (4 out of 18), numbers and patterns. Before the course, students mentioned algebra (10 out of 20), numbers (4 out of 20) and statistics-probability (5 out of 20). After the course, students mentioned mainly algebra, statistics-probability, and numbers. It should be noted that 3 students stated that FT can be developed in all domains of mathematics.

Table 4

		Teachers		Students	
		Before (out of 20)	After (out of 18)	Before (out of 20)	After (out of 17)
1	Algebra	10	9	10	9
2	Geometry	1	4	2	3
3	Numbers	2	2	4	6
4	Statistics-Probability	2	4	5	7
5	Patterns	-	2	-	-
6	Functions	-	-	2	3
7	Problem Solving	2	1	1	-
8	All domains	-	-	-	3

5. What do you consider to be a goal when teaching about functions? Expand your answer.

Table 5 presents the goals of teaching about functions mentioned by teachers and students, before and after the course. Before the course, teachers mentioned a variety of goals, such as understanding phenomena of everyday life and social life (3 out of 20), recognizing patterns, structures and rules (3 out of 20). After the course, most of the students stated as the goal of teaching about functions developing an understanding of dependencies and relationships (10 out of 18). In respect to students, most of them referred to developing an understanding of dependencies and relationships before and after the teaching course (9 out of 20 before and 11 out of 17 after).

Table 5

		Teachers		Students	
		Before (out of 20)	After (out of 18)	Before (out of 20)	After (out of 17)
1	Develop an understanding of dependencies and relationships	2	10	9	11
2	Understand phenomena of everyday life and social life	3	-	1	3
3	Develop functional thinking	1	2	-	2
4	Foster problem solving strategies	2	-	-	-
5	Develop an understanding of handling functions	1	-	3	-
6	Gain understanding of reciprocal relationships of numbers	2	-	1	1
7	Students use different representations of functions	1	-	-	-
8	Recognize patterns, structures and rules	3	3	1	4

6. Brady Problem

In the Brady Problem, participants were asked to solve the problem (provide the number of persons for 8 and 20 tables) and then suggest questions that they could pose to students based on the task.

Table 6 provides the number of teachers and students that solved the problem correctly and the strategy they used. In respect to teachers, 18 out of 20 teachers correctly solved the problem before the course, while all of them did so after the course. Before the course, 11 out of the 20 teachers used a correspondence general strategy (provided the general rule) and 5 of them a correspondence-particular one (explained how to calculate the number of persons for the given number of tables). After the course, 15 out of the 18 opted a correspondence general strategy.

Sixteen out of the 20 students correctly solved the problem before the course and all of them did so after the course. In respect to the strategy used, 12 of them used a correspondence-particular strategy and 6 a correspondence general strategy before the course. After the course, 6 of them used a correspondence-particular strategy and 10 of them preferred a correspondence general one.

Table 6

Teachers				Undergraduate Students			
Before (out of 20)		After (out of 18)		Before (out of 20)		After (out of 17)	
Correct.	Strategy	Correct.	Strategy	Correct.	Strategy	Correct.	Strategy
Wrong 2	Corresp. Particular 5	Wrong 0	Corresp. Particular 2	Wrong 4	Corresp. Particular 12	Wrong 0	Corresp. Particular 6
Correct 18	Corresp. General 11	Correct 18	Corresp. General 15	Correct 16	Corresp. General 6	Correct 17	Corresp. General 10

No evidence 4	No evidence 1	No evidence 2	No evidence 1

Table 7 gives insight, which types of questions were suggested by the teachers. Before the course, they posed mainly a question requiring the general term (8 out of 20), a far-transfer term (5 out of 20) and a near term (3 out of 20). After the course, a larger number of respondents referred to the different conceptions of function (i.e., extend a pattern, find the covariation rule, or find the correspondence rule). In particular, 10 teachers posed a question asking for the general term, 7 teachers posed a far-transfer term question, 6 posed a find near term question, 4 posed a covariation question and 8 of them provided a question that gives the number of people and asks for the number of tables.

Before the course, students provided questions that required finding a far-transfer term (7 out of 20), finding the general term (5 out of 20) and finding the number of tables given the number of people (8 out of 20). Again, after the course, students posed questions that captured different conceptions of function and modes of functional thinking. Students posed questions that required finding a near term (6 out of 17), a far-transfer term (10 out of 17), covariation (5 out of 17), general term (12 out of 17) and a problem requiring the number of tables, given the number of people (8 out of 17).

Table 7

	Type of question	Teachers		Students	
		Before	After	Before	After
1	Find near term	3	6	1	6
2	Find far-transfer term	5	7	7	10
3	Covariation question		4		5
4	Find the general term or provide the rule	8	10	5	12
5	Formulate a problem - Provide the number of people and ask for the number of tables	1	9	8	8
6	Formulate a problem with remainder	1	1		
7	Formulate a problem after calculating the number of people for a table			2	1
8	Change the rule of the pattern	1		3	
9	Change the arrangement of the tables or the sitting rule	1		1	
10	General questions: what numbers and variables are involved, explain your thinking	1	2	1	2
11	Ill-defined problems	1		2	
12	Problems that do not relate to functional thinking	1		1	

7. Feedback Questionnaire

Participants' responses to the feedback questionnaire were extremely positive. Table 8 presents the mean and standard deviation of their responses to the feedback questions. Their mean score was greater than 4.69 (out of 5) in all questions.

Participants answered that what they liked most in the course was the embodied nature of the activities, the links between theory and practice, their involvement in interactive digital activities and the fact that they first acted as students and then discussed the teaching approaches and practices.

Most of the participants (90%) stated that they would not change anything in the structure and the content of the course. Few of them suggested that the duration of the course could be longer and include co-teaching of modules in their classrooms.

Finally, participants claimed that there is a need of training courses for other aspects of algebraic thinking. They suggested that the research team could organize teacher courses on how to integrate digital tools in mathematics teaching, with an emphasis on practical implications and teaching practices for orchestrating mathematical discussions.

Table 8

	Mean	**SD**
I learned interesting things during the teacher training course about mathematics.	4.96	.20
I learned interesting things during the teacher training course about teaching.	4.96	.20
The knowledge I gained is useful for my professional development.	4.96	.20
The teacher training course, as implemented, was appropriate and effective.	4.96	.20
The knowledge I gained is useful for teaching Mathematics.	4.69	.47
I will use the teaching material developed by the project in my teaching.	4.77	.58
I will use the digital tools developed by the project in my teaching.	4.88	.33
The digital tools of the project are interesting.	4.92	.28
The digital tools of the project facilitate the development of conceptual understanding of mathematics.	4.88	.43
I would recommend this training course to a colleague of mine.		

Discussion

The results of this study provide compelling evidence that in-service, pre-service teachers, and undergraduate students' participation in the training course changed their conceptions' regarding FT in respect to its aspects, relevant reference domains, appropriate age groups and goals of teaching. Before the course, most of teachers and students stated that they have never heard before the term FT. After the course, teachers and students referred to specific conceptions of functions (input-output, covariation, correspondence), the importance of FT in different domains of mathematics, type of activities that facilitate FT development, such as patterning activities, and concepts that relate to function. Further, after the course, they named as FT key words the notions of relations, input/output, covariation and correspondence. In respect to the appropriate age, teachers and students stated that FT can be developed with appropriate activities from kindergarten or Grade 1. Furthermore, they claimed that FT is relevant with most of mathematics topics, such as algebra, geometry, numbers, and statistics-probability. They also stated that the goal of teaching about functions should be to develop an

understanding of dependencies and relationships that relates to specific aspects of function. Teachers and students responded adequately to a patterning problem that requested finding far-transfer terms by applying correspondence particular or correspondence general strategies. In terms of pedagogical applications, students posed questions taking into consideration progressive levels of functional thinking, such as finding near terms, far-transfer terms, covariation and correspondence rules and problem-solving extensions.

In conclusion, our study has shown promising results for the potential of the teacher training course to change teachers' and students' conceptions of FT and facilitate the development of a theoretical and empirically based pedagogical content knowledge for FT. The feedback provided by the participants also showed their active engagement in exploratory teaching practices that build on the effective exploitation of embodied experiences and digital tools.