Report on implementation phase in Poland

Modules:	Cooling of water, Embodying graphs, Filling vessels
Responsible Partner:	Pedagogical University of Krakow
Grade Level/Age Range:	Grades 7,8,9
Sample size:	65 (grades 7-8), 89 (Grade 9)
Brief Description of Testing / Intervention:	There were two phases – pilot implementation in Grades 7-8 and main implementation in Grade 9. Pilot phase consisted of three learning environments (3 x 45 minutes) and main implementation consisted of two 45-minutes lessons intervention. It included the following modules: Embodying graphs and Cooling of water (and for the pilot phase: Filling vessels). Each module had 1 lesson. The lessons conducted as part of the pilot implementation phase took place in three classes (7a, 7b, 8a) consecutively with 1 or two weeks of break. The lessons conducted as part of the main implementation phase took place in three classes (9 grade) of Technical High School consecutively with 1 or two days of break. Lesson 1: **Cooling of water** Students predicted, verified, created, and analysed graphs of temperature changes in time on the basis of performing physical experiments during the lesson. Lesson 2: **Embodying Graphs** Students created graphs through embodied experiments - using hand movement and their smartphones with PhyPhox software installed. -They created and observed many functions with different graph shapes, -They 'produced' movement according to the given graph, -They also interpreted and worked with graphs of various continuous functions. During the lesson students discovered - at the pre-definition stage in practice - the conditions for defining the concept of a function Additionally, during pilot phase we implemented: Lesson 3: **Filling vessels** Students used real experiments to investigate how the filling process differs in the graph for different vessels.

Implementation phases general description:

The implementation was carried out in two phases – pilot implementation at a primary school level and implementation at the secondary school level.

I) Primary school – pilot implementation

The pilot implementation was carried out with students of three classes between 13 and 15 years old: two classes of grade seven (7a & 7b) and a one class of grade eight (8a) from Primary School No. 37 in Krakow with a presence of a mathematics teacher and a physics teacher.

The implementation phase took place in the days 10.05.2022 – 09.06.2022 and included teaching of three learning environments: Cooling of water, Embodying graphs and Filling vessels. There were 65 students taking part in the three learning environments.

The implementation was done to pilot the learning environments in order to improve them, check how much time they require, whether they are feasible at primary school level and whether they are effective. No tests were carried out during this implementation phase, the feedback was only based on the observation of the classes and oral interviews with the teachers.

The phase allowed the research team to improve the learning environments and precisely describe them. The implemented learning environments proved to be relevant and suitable for primary school.

II) Secondary school – main implementation

The teaching experiment was carried out with students of a grade nine (14-15 years old) from a Mechanical Engineering Technical High School in Krakow (secondary school level – a technical school). The starting point and preliminary knowledge of the students were: having experience with using the coordinate system, working with tables, forming an addition or subtraction expression to represent the relationship between two quantities; using a ratio to describe the relationship between two quantities; having partial experience with reading from a graph.

The implementation phase took place in the days 28.02.2023 – 02.03.2023 and included the following:

- PRE1 - pre test (assessments item UP – Functional relationships in graphs),
- PRE2 - pre-test (common FunThink part, 3 items),
- teaching - two LE: Cooling of water & Embodying Graphs (28.02.2023 – 02.03.2023)
- POST1 – post test (assessments item UP – Functional relationships in graphs),
- a questionnaire about what students liked and disliked while working in the classroom with the two learning environments,
- POST 2 - post-test (common FunThink part, 3 items).

There were 89 students who completed PRE1 and POST1 tests before and after the experiment.

There were 23 students who completed both PRE2 and POST2 tests before and after the experiment. Due to the fact that each of the PRE2 and POST2 tests took one lesson unit, i.e. 2 x 45 min in total for every class, the school did not agree to conduct them in all 3 classes dedicating 6 lesson hours to the evaluation itself. For this reason, this phase of the evaluation was only tested in one class (1Td) consisted of 28 students. 5 students were excluded from the summary due to their absence from at least one of the two tests. Therefore, a specially designed additional short PRE1 and POST1 tests were performed in all classes.

The teacher who led the lessons had been teaching for 5 years. She collaborated with the research team.

PRE1 and POST1 tests: assessments item UP – Functional relationships in graphs

Methodology:

<table>
<tr><td>Grade Level:</td><td colspan="4">Grade 9</td></tr>
<tr><td>Module Correspondence:</td><td colspan="4">Cooling of water, Embodying graphs</td></tr>
<tr><td rowspan="4">Functional Thinking:</td><td>Input – Output</td><td></td><td></td><td></td></tr>
<tr><td>Covariation</td><td></td><td></td><td></td></tr>
<tr><td>Correspondence</td><td></td><td></td><td></td></tr>
<tr><td>Object</td><td></td><td></td><td></td></tr>
<tr><td>Learning Goals being Assessed:</td><td colspan="4">✓ Interpret graphs that represent co-varying quantities (temperature in time)
✓ Distinguish functional from non-functional relationships (continuous function)</td></tr>
<tr><td>Brief Description:</td><td colspan="4">✓ A real life context is provided and students have to select the graph that corresponds to the situation
✓ Students have to decide which graph can describe the temperature change over time (distinguish functional from non-functional relationships in a real-world context)</td></tr>
<tr><td>Cognitive Domain:</td><td colspan="4">Applying & Reasoning</td></tr>
</table>

The following Table 1 presents PRE1 and POST1 test items:

<table>
<tr><th>PRE1 test</th><th>POST1 test</th></tr>
<tr><td colspan="2">Question: Which of the figures could represent a graph of temperature change over time? Decide YES/NO and justify your answer.</td></tr>
</table>

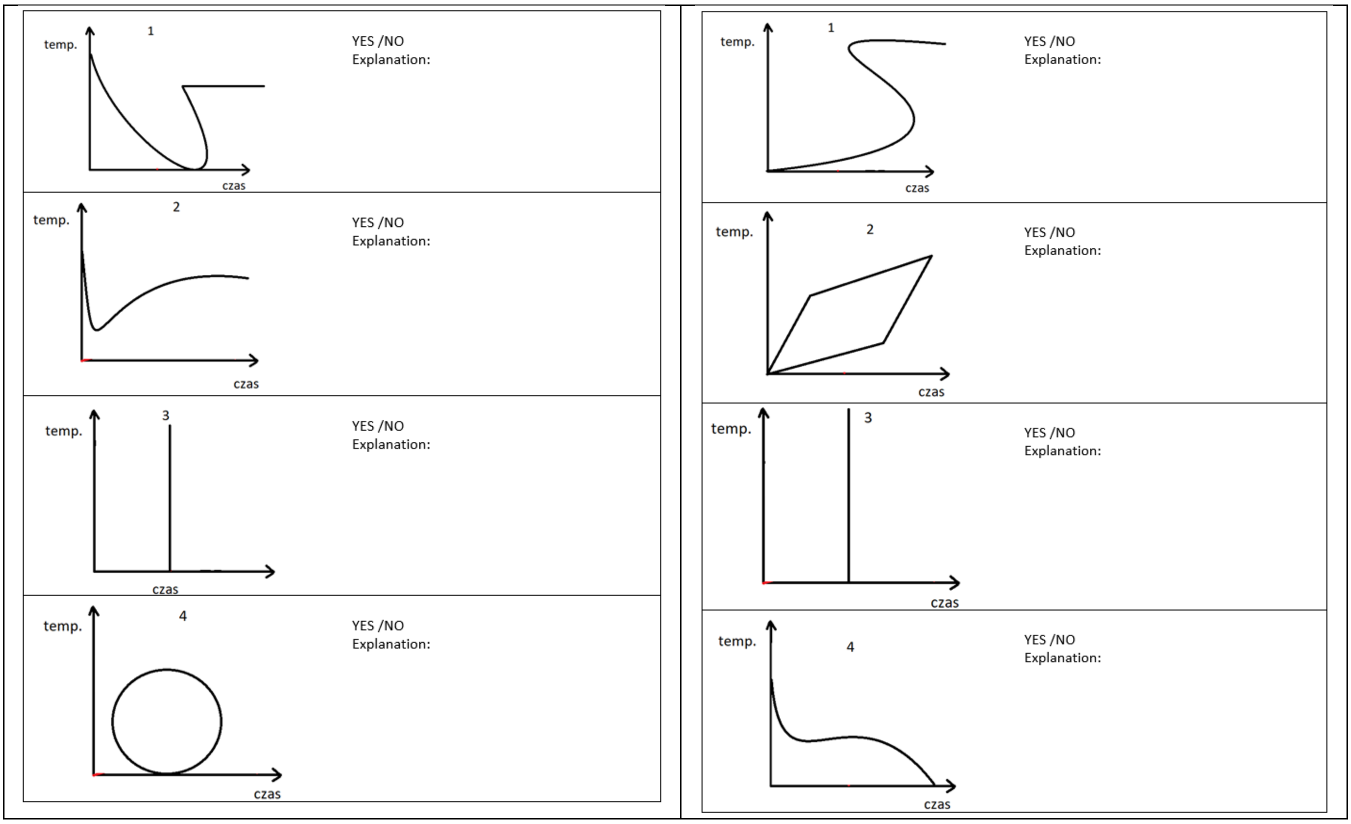


Table 2 shows the characteristics of the graphs chosen for the PRE1 and POST1 tests.

Table 2. Characteristics of the graphs

characteristics of the graphs	Task no. in the test	
	PRE1	POST1
Going back in time, curve - not a function	Figure 1	Figure 1
Graph of a function	Figure 2	Figure 4
Vertical straight line (not a function - infinitely many values for one x)	Figure 3	Figure 3
Geometric shape (circle, rhombus - not a function - two values for one x)	Figure 4	Figure 2

Students were to diagnose and justify which graph can represent a graph of temperature change over time – therefore the question was on intuitive understanding of function definition and justification why the provided graph can or cannot represent a function.

Three different groups of students (class teams) were surveyed, consisting of: 28, 29 and 32 people, respectively, which is a total of 89 people.

It should be noted that this group included 5 students from Ukraine who had additional difficulties related to the lack of knowledge of the Polish language and difficulties in understanding the content of the instructions.

Due to the different number of people in classes, relative percentages of results obtained in the three groups were calculated for analysis and comparison.

1 point was awarded for a correct answer. Justifications for the choice of answers were also assessed by awarding half values of points in the case of incomplete justification.

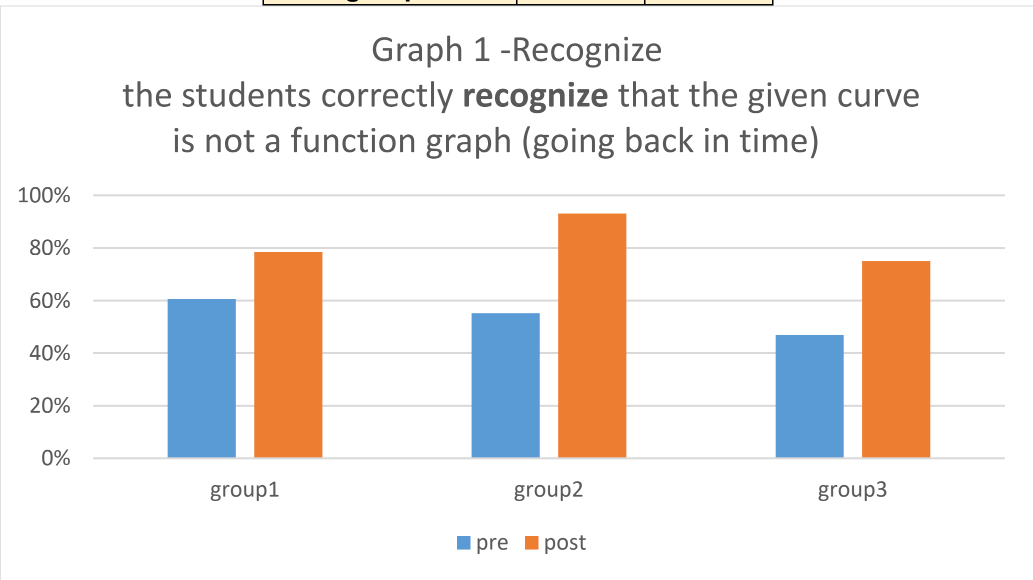
Total possible score for the PRE1 and POST1 tests is 8 points:

(a) The possible graph – answer YES (1 point)
(b) Impossible graphs– answer NO (1 point in every case)
(c) Explanation 1 points for each explanation – partial credit was possible: 0,5 point

Results:

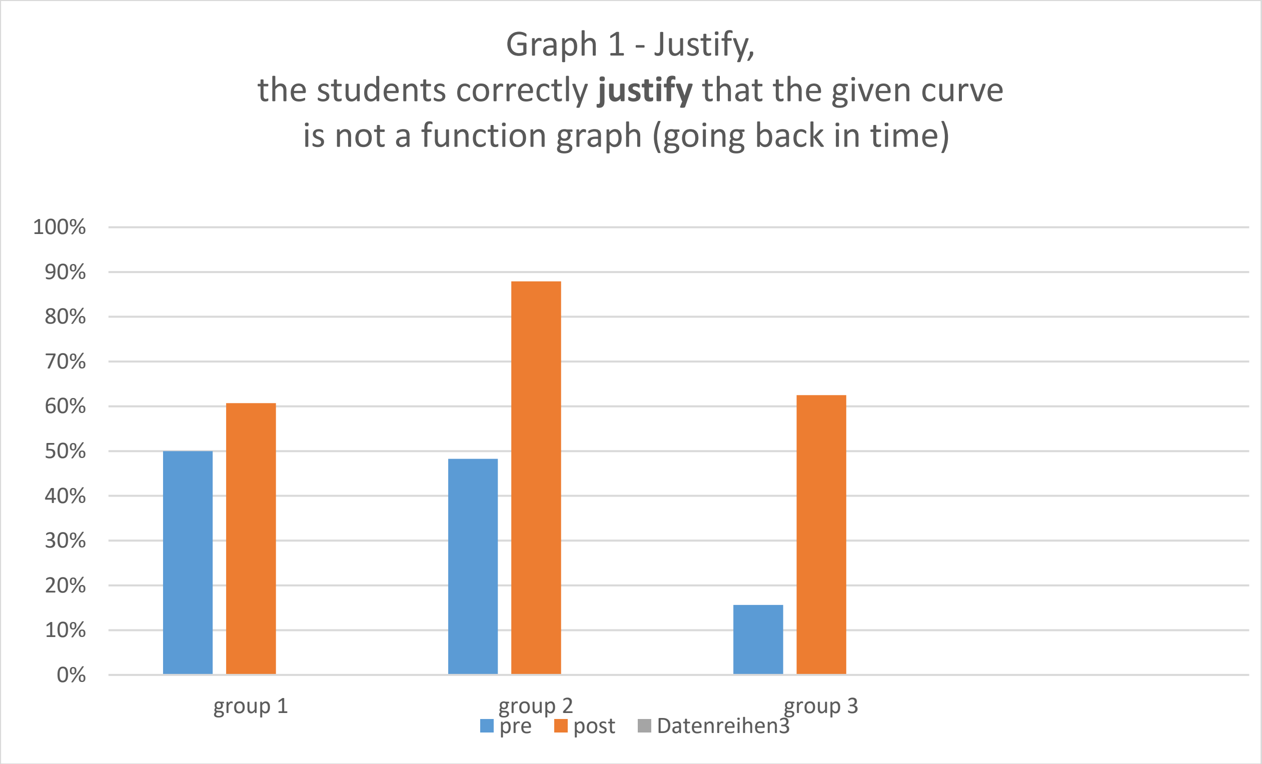
Graph 1 Recognize. The students correctly **recognize** that the given curve it is not a function graph (going back in time)

	pre	post
group1	61%	79%
group2	55%	93%
group3	47%	75%



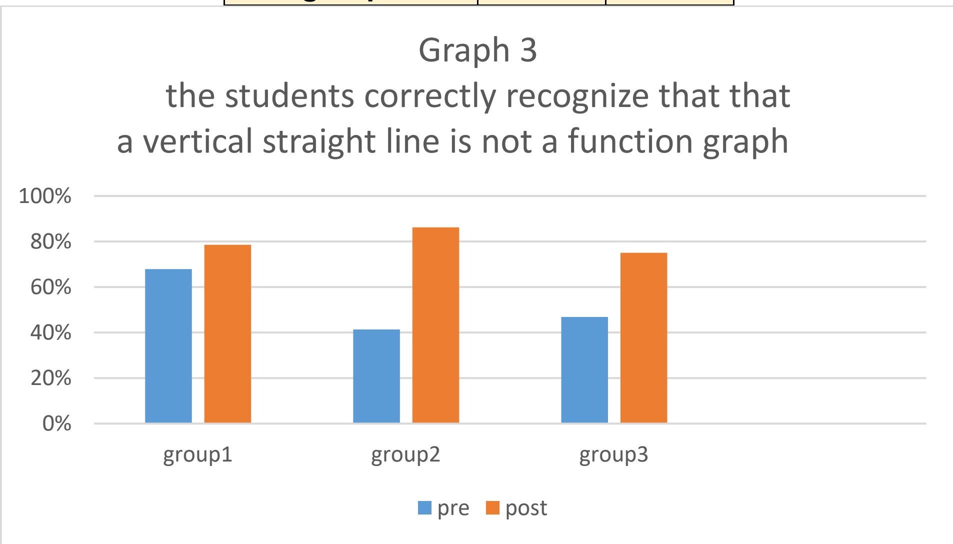
Graph 1 Justify. The students correctly **justify** that the given curve it is not a function graph (going back in time)

	pre	post
group 1	50%	61%
group 2	48%	88%
group 3	16%	63%


Graph 1 - Justify,
the students correctly **justify** that the given curve
is not a function graph (going back in time)
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
group 1
group 2
group 3
pre
post
Datenreihen3

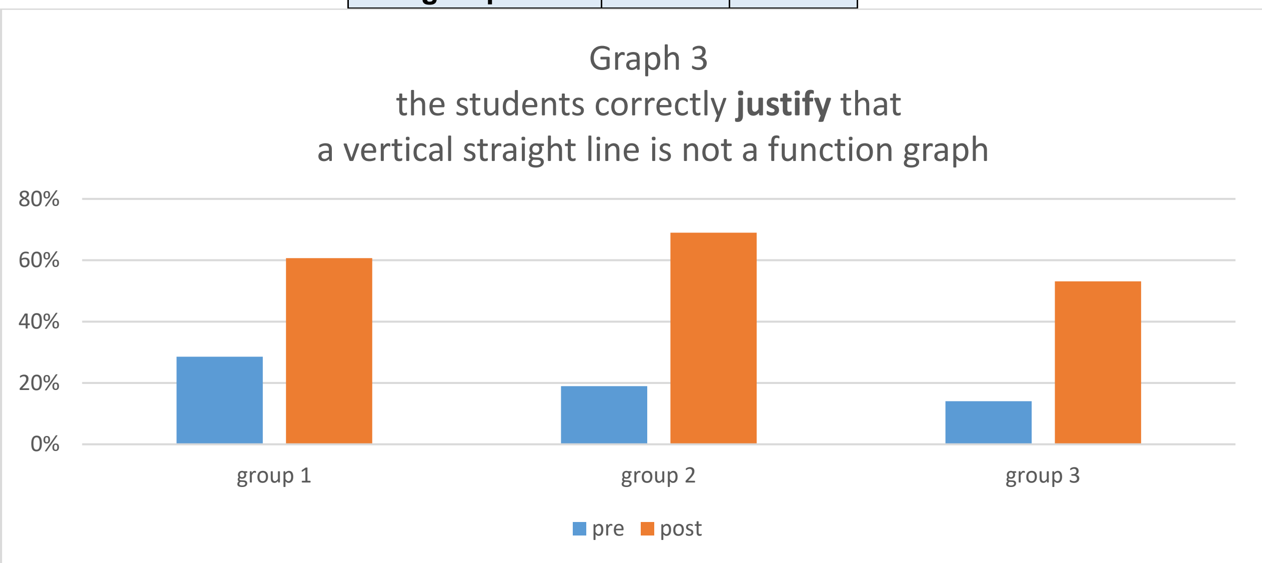
Graph 3 Recognize. The students correctly **recognize** that a vertical straight line is not a function graph

	pre	post
group1	68%	79%
group2	41%	86%
group3	47%	75%



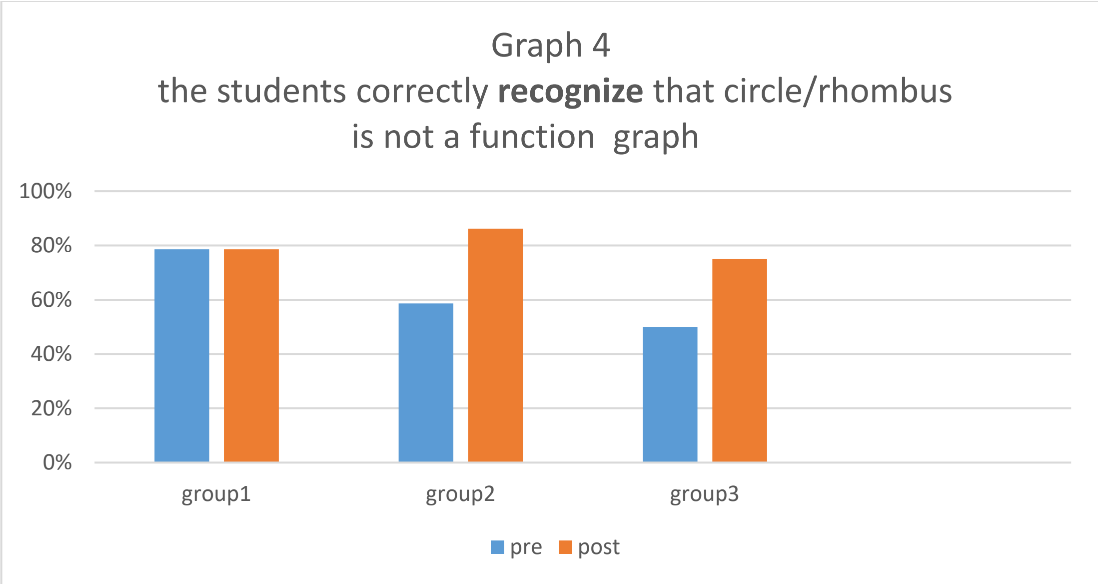
Graph 3 Justify. The students correctly **justify** that that that a vertical straight line is not a function graph

	pre	Post
group 1	29%	61%
group 2	19%	69%
group 3	14%	53%



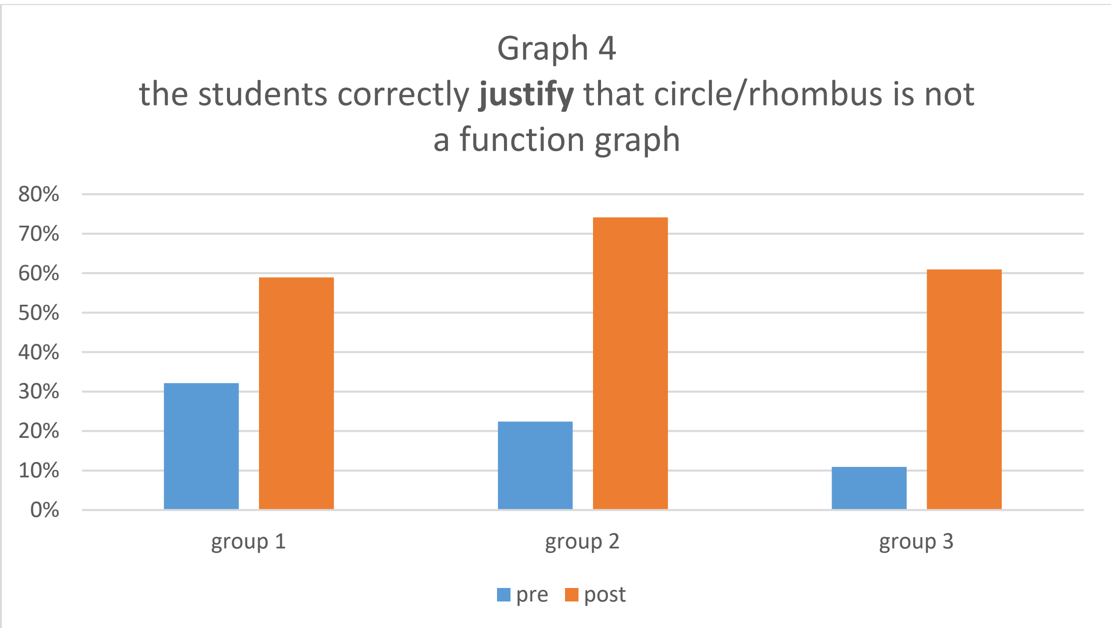
Graph 4 Recognize. The students correctly **recognize** that that circle/rhombus is not a function graph.

	pre	post
Group 1	79%	79%
Group 2	59%	86%
Group 3	50%	75%



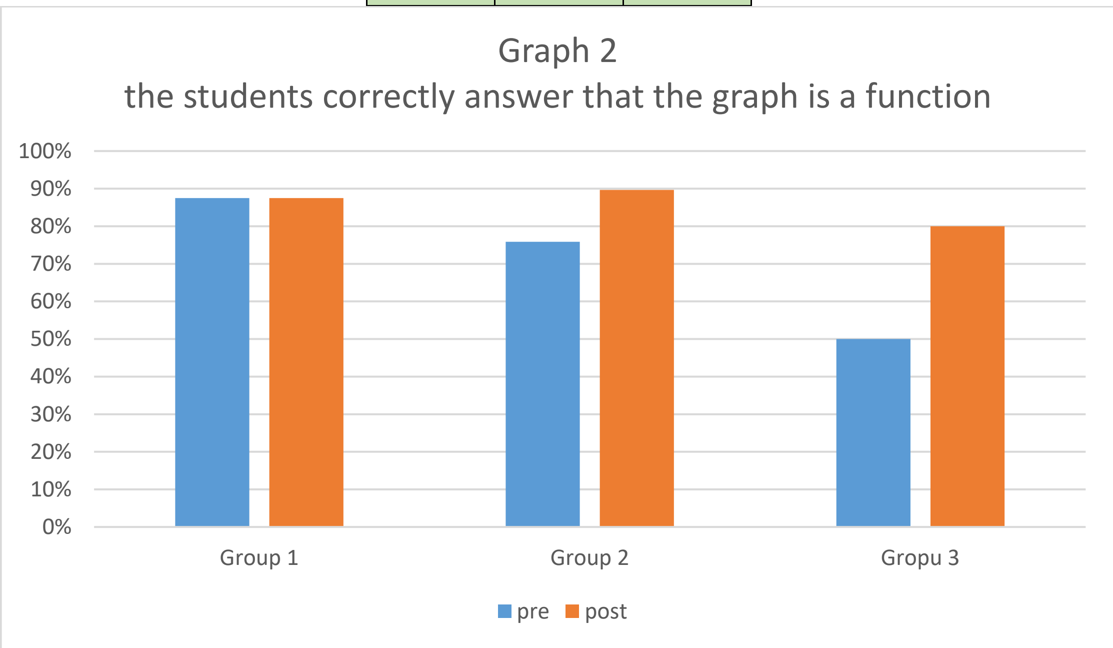
Graph 4 Justify. The students correctly **justify** that circle/rhombus is not a function graph.

	pre	post
group 1	32%	59%
group 2	22%	74%
group 3	11%	61%



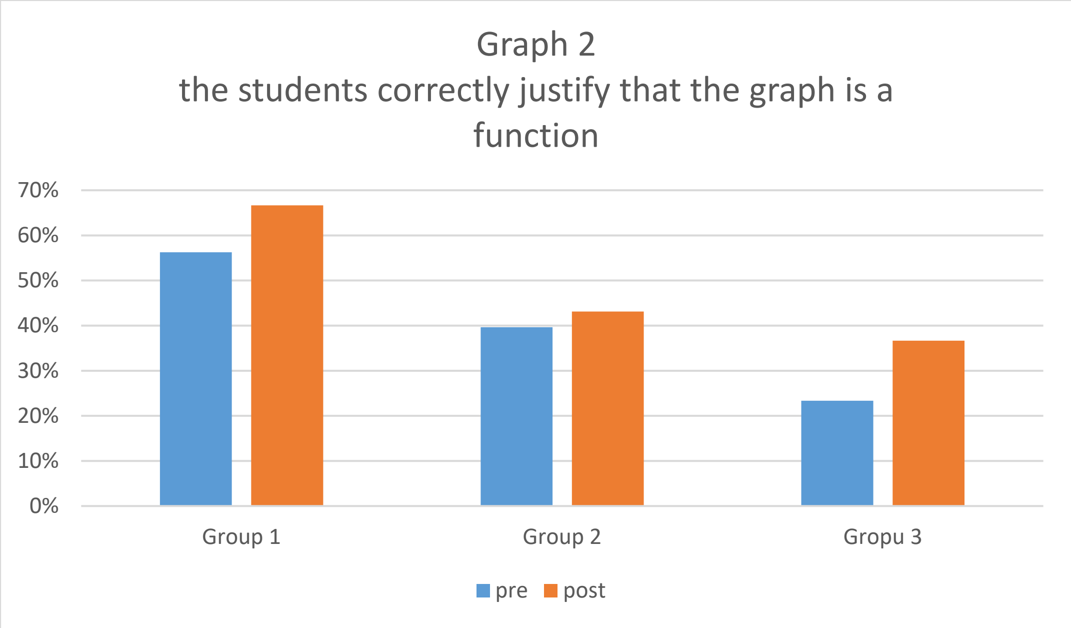
Graph 2 Recognize. The students correctly recognize that the graph is a function.

	pre	Post
Group 1	88%	88%
Group 2	76%	90%
Group 3	50%	80%



Graph 2 Justify. The students correctly justify that the graph is a function

	pre	post
Group 1	56%	67%
Group 2	40%	43%
Group 3	23%	37%



The improvement in student performance on POST1 tests compared to the PRE1 test demonstrates the great potential of the used learning environments to develop functional thinking after only 2 hours of intervention.

PRE2 and POST2 tests: common FunThink part

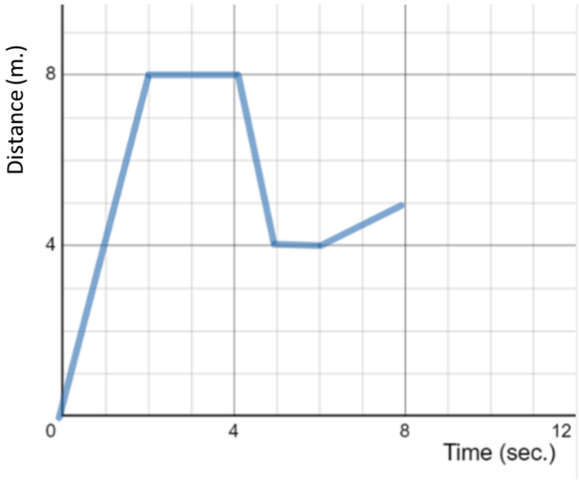
Methodology - Testing items and scoring

PRE2 and POST2 tests were identical, consisted of three items.

Item 1 (see Figure 1) was based on an item used by Duijzer (2020) and measured graph interpretation and construction. Students observe a graph with data about a travelling car (distance-time). The first question required global and local interpretation of the graph as students had to identify which parts of the graph represented moving away or moving towards a person. The second question asked to identify when the car moves the fastest. The third question asked students to extend the graph for the following seconds based on a given description.

Figure 1

Ann plays with a remote-control car toy. The following graph presents the distance of the car from Ann in respect to time.



a. When was the car moving away from Ann and when towards Ann? Please explain.

b. When did the car move the fastest? Please explain.

c. Complete the graph for the next four seconds based on the following:

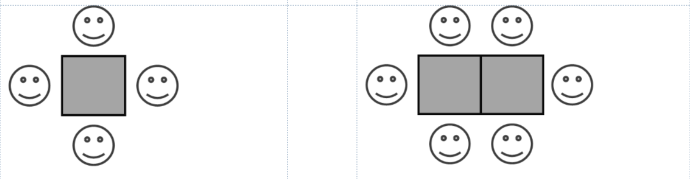
"The car moved away from Ann for another one second and then moved towards her, without reaching her."

Item 2 (see Figure 2) was based on the Birthday Party item that was used by Blanton et al. (2015). The first question of the item required to find a term of a pattern that could be calculated based on a recursive, covariation or correspondence rule. The second one required calculating a far-transfer item and the third one asked student to provide the general rule of the pattern.

Figure 2

Brady is having his friends over for a birthday party. He wants to make sure he has a seat for everyone. He has square tables.

He can seat 4 people at one square table in the following way:		If he joins another square table to the first one, he can seat 6 people:

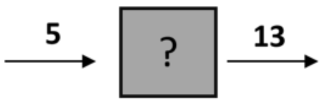


If Brady has 8 tables, how many people can he seat at his birthday party? And how about 20 tables? Can you find a rule that describes the relationship between the number of tables and the number of people who can sit at the tables?

Item 3 (see Figure 3) was developed for the purpose of the study based on ideas suggested by Pittalis et al. (2020) and Ng (2018) and measured students' ability to identify the numerical relation between two sets of values to find the input or output value of a function machine. Students were also asked to express the rule of the machine using symbols.

Figure 3

a. Find below a function machine. A number is entered, and the machine gives an output value based on a secret rule.



The table shows some inputs and outputs of this machine. Complete the empty cells. Show your calculations in the last column.

INPUT	OUTPUT	CALCULATIONS
0	3	
5	13	
7	17	
10	23	
12		
15		
	11	
	43	

b. John entered the symbol * in the machine. What will be the output? Please explain.

Results and Discussion:

Results

The table below presents the result of 1a item in three categories: correctness, reasoning and strategy.

Item 1a - correctness	1a_PRE	1a_POST
0 = wrong	9	3
0,25 = one part away	3	7
0.5 = two parts away OR the part towards her correct	0	2
0.75 = one part away AND one part towards	2	3
1 = both parts correct	6	5
99 = NA	3	3
Item 1a - reasoning	**1a_ PRE**	**1a_POST**
10 = single variable reasoning	2	0
20 = multiple variable reasoning (2)	4	1
21 = multiple variable reasoning (3)	1	2
22 = object view	0	1
99 = "no idea" or "?"	16	19
Item 1a - strategy	**1a_ PRE**	**1a_POST**
0 = no intervals, points	3	1
0.5 = at least one interval/ some but not all	10	11
1= All intervals	5	5
99 = NA	5	6

The responses to Task 1a did not show any significant improvement in students. There was only a slight reduction in the number of incorrect answers and single variable reasoning in the post-test.

The next table presents the result of 1b item also in three categories: correctness, reasoning and strategy.

Item 1b - correctness	1b_PRE	1b_POST
0 = wrong	12	3
0.5 = one interval correct	5	14
1 = both intervals correct	3	4

99 = NA	3	2
Item 1b - reasoning	1b_PRE	1b_POST
10 = single variable reasoning	2	1
21 = multiple variable reasoning (3)	3	3
22 = object view	2	3
99 = "no idea" or "?"	16	16
Item 1b - strategy	1b_PRE	1b_POST
0 = no, points	6	1
0.5 = at least one/ some but not all	8	11
1 = all intervals	3	4
99 = NA	6	7

The responses to Task 1b shows a significant improvement in correctness at the lowest level. The number of 12 wrong answers in pre-test decreased to 3 in post-test, and the number of at least one correct interval increased from 5 in pre-test to 14 in post-test. The category "reasoning" does not show any significant improvement, but again in category "strategy" we can observe an improvement - the number of 6 only point-oriented answers (without any interval) in pre-test decreased to 1 in post-test, and the number of at least one correct interval increased from 8 in pre-test to 11 in post-test.

Responses to item 1c show no significant improvement, which is shown in the next table.

Item 1c	1c_PRE	1c_POST
0 = wrong	5	4
0.5 = one part correct	3	2
0.75 = line goes downward but crosses x-axis	0	0
1 = both parts correct	11	11
99 = NA	4	6

The next table presents the result of item 2 in three categories: correctness, reasoning and final rule.

Item 2 - correctness	2_PRE	2_POST
0 = wrong	1	0
0.5 = only the answer for 8 or 20 tables is correct	4	1
1 = both answers are correct	18	18

99 = NA	0	4
Item 2 - reasoning	2_PRE	2_POST
Correspondence	3	9
Covariational	1	2
Recursive/ variational	19	12
Item 2 - rule	2_PRE	2_POST
0 = no correct rule	11	4
0.5 = a rule but not the relation between tables and people	5	5
1 = correct rule	5	12
99 = NA	2	2

Analysis of Task 2 shows a significant progress in the students' solutions. It can be seen in the types of reasoning revealed by the students and in the increased number of correct formulas. The number of reasonings based on correspondence increased threefold (from 3 to 9) and the number of reasonings considering only one variable decreased from 19 to 12. As for rule correctness, it increased from 5 to 12.

The next tables presents the result of item 3 in three categories: input, output and final rule.

Item 3 - input	3_PRE	3_POST
0 = wrong	17	2
0.5 = one of the two input values correct	3	0
1 = both values correct	3	16
99 = NA	0	5

Item 3 - output	3_PRE	3_POST
0 = wrong	20	2
0.5 = one of the two output values correct	0	1
1 = both values correct	3	15
99 = NA	0	5

Item 3 – reasonings & strategy (rule)	3_PRE	3_POST
10 = Correspondence particular	1	1
11 = Finding the rule based on covariation	0	0
12 = Any other correct strategy	0	0
20 = Correspondence general	1	9
21 = Correspondence general with a mistake	0	3
22 = Object view	1	0
70 = Variation - only one quantity	1	1
71 = Repeat the pattern of the given examples	1	0
72 = Grasp the correspondence rule only in particular examples	0	0
73 = Identification of different additive rules	0	2
80 = Random numbers without showing that they identified a relation	7	0
81 = Correct completion of the table without showing calculations	11	7

An analysis of the Item 3 results shows great progress by the students in this task.

The number of incorrect answers has significantly decreased: for input from 17 to 2, for output from 20 to 2; consequently, the number of both correct answers has increased - in the input category from 3 to 16 and in the output category from 3 to 15.

The last table also shows an increase in the reasoning – the number of the most important category 20/21 showing reasoning as a general correspondence rule increased from 1 to 12. Moreover the category "80 = Random numbers without showing that they identified a relation" disappeared completely in the post-test.

Summary:

The results of this study provide evidence that the proposed learning environments contribute to the development of students' functional thinking. The improvement in student performance on the post-tests compared to the pre-tests demonstrates the potential of the used learning environments to develop functional thinking. This is particularly evident in the first items (PRE1 and POST1) – that were used immediately before and after the lessons.

The second test was done with a smaller sample, so the results are less spectacular, but show great progress in the patterning tasks and with the function machine. In addition to the improvement in students' performance, the analysis also showed a change in the strategies used by the students. For example, in items 2 and 3 of the pre-test, students focused mainly on filling in the values (usually using recursive strategies to determine the relationship between the variables involved), whereas in the post-test, more students used more advanced strategies such as correspondence or covariation.

It turns out that the most difficult item for the students was Item 1. When working on sub-items 1b, 1c, the students also revealed misconceptions - treating the graph as a picture, the trajectory of the movement. Surely this issue should be given more attention in the future.

The individual learning environments respected the design principles to foster functional thinking as described in the Vision document. From the students' responses to the questionnaire administered at the end of the intervention, we conclude that the success of the intervention is related to the phenomenologically rich situations and to the linking of the embodied experience of the digital-embodied tasks, which provide a concrete experience in combination with the abstract nature of functional thinking.

In conclusion, our study has shown promising results in the use of the mentioned learning environments to support functional thinking. These results open the way for further research in this area and for the development of further innovative, effective instructional designs in mathematics education.