

## Report on teacher testing in Poland

<b>Items used:</b>	<p>Pre- and post- tests:</p> <ul style="list-style-type: none"> <li>- Functional thinking definition</li> <li>- Grades and topics appropriate for FT development</li> <li>- Goals of teaching about functions</li> </ul> <p>Likert questionnaire of course assessment (11 items) Open questions questionnaire of course assessment (3 items)</p>
<b>Responsible Partner:</b>	UP Krakow, Poland
<b>Background of the participants:</b>	pre-service teachers in mathematics in their 4th year of mathematics for teachers study
<b>Sample size:</b>	10
<b>Brief Description of Testing:</b>	<p>Pre-test</p> <p>Long course</p> <ul style="list-style-type: none"> <li>- 30 hours</li> <li>- 135 minutes long sessions (3h)</li> <li>- modules used and their allocations: Introduction and pre-assessment (3h), Knowledge of Topic (3h), Function and its representations (3h), Functional Thinking (3h), Design principles (3h), Learning environments (11h), Curriculum (1h)</li> </ul> <p>Post-test</p> <p>Final meeting</p>

### Method:

Pre-test and post-test took place during the semester. 10 pre-service teachers participated at both of them. The pre-service teachers were informed about the developmental goals of these tests, and they were not used for scoring and grading pre-service teachers. The results were coded in alignment with the codebook developed by the FunThink Team. The main interest in testing is not how many times a given code occurs for a particular pre-service teacher, but whether it occurs at all. Thus, the numbers in the following tables reflect the number of pre-service teachers for whom the code occurred.

After finishing the course, also an open questionnaire of course assessment was completed by course participants in a written, paper-pencil form. The questionnaire was conducted in an anonymous form, which allowed participants to have a free, objective and also critical view of the whole course.

The questionnaire contained 11 closed questions of the evaluation type, in which participants were asked to rate how much they agreed with a given statement by selecting an answer on a scale from 1 to 5, where 1 meant full disagreement and 5 meant full agreement.

This material is provided by the [FunThink Team](#), responsible institution: Pedagogical University Krakow



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Please rate the following statements in context of the teacher training course. (1 – Absolutely Disagree / 5 – Absolutely Agree):

- |   | <b>1</b>              | <b>2</b>              | <b>3</b>              | <b>4</b>              | <b>5</b>              |
|---|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 1. I learned interesting things in the teacher training course about mathematics.                         | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 2. I learned interesting things in the teacher training course about teaching.                            | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 3. The knowledge I gained is useful for teaching Mathematics.   | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 4. The structure of the teacher training course was appropriate and effective.                            | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 5. The content of the teacher training course was appropriate and effective.                              | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 6. The way of delivery of the teacher training course was appropriate and effective.                      | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 7. I will use the material of the project in my teaching.   | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 8. I will use the digital tools of the project in my teaching.  | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 9. The digital material of the project is interesting.  | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 10. The digital material of the project facilitates conceptual understanding of the mathematics concepts. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 11. I would recommend this seminar to a colleague of mine.  | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

Please answer the following questions.

1. What did you like best about this teacher training course?
  
  
  
  
  
  
  
  
  
  
2. What suggestions do you have for improving this teacher training course?
  
  
  
  
  
  
  
  
  
  
3. What other suggestions would you like to share with us regarding this teacher training course?

**Thanks for completing this questionnaire!**

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Results:

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**Question: Have you ever encountered the term functional thinking? If so, what do you consider functional thinking to be? If not, what do you expect it to be? Expand your answer.**

**Results Pre-test:**

In the pre-test, all of the pre-service teachers (n=10) were unfamiliar with the concept of functional thinking.

The majority of them (n=6) associated it with logical thinking, abstract thinking or thinking that can apply learned knowledge in practice, based on linguistic connotation in Polish that something works ("functions").

Responses:

- *I thought it was about thinking according to a pattern, a way of thinking, something to make it working (functioning)*
- *I suppose functional thinking is such analytical, insightful thinking, associated with combining various facts.*
- *To me, Functional Thinking can mean noticing various relationships between things or activities in everyday life on a similar basis to how we deal with functions for math. (After all, we focus on them quite a bit, discuss their properties deeply, etc.).*
- *Knowing this may be about forming students' intuition for solving mathematical problems on a general scale, rather than just thinking about a slice of the problem (as the graph shows a fragment)*
- *Ability to go beyond thinking very concretely about selected numbers /objects, to adapt to more abstract thinking*
- *I associate it with thinking that can be applied to many problems in life, we think functionally so as to deal with the task problem encountered.*

A minority (n=4) mentioned a notion of function in their guessing, but either giving very general description - thinking about functions or focusing on solving tasks and getting knowledge about the function notion.

**Results Post-test:**

The following table shows in details which aspects of the function and functional thinking were spontaneously mentioned by the course participants. This table shows that there was a huge progression between the results of the pre-test, where no one knew the term "functional thinking", and the post-test, where all aspects of the function concept were spontaneously mentioned by at least four people.

Half of the pre-service teachers spontaneously stressed the necessity of using and teaching about functions in order to model reality (n=5), and only some pre-service teachers (n=4) stuck to the very general description.

	Input-Output	Covariation	Correspondence relation	Object	Modelling	General thinking
KH	✓		✓	✓		
SP	✓		✓	✓		
KS	✓	✓	✓	✓		
IO		✓	✓	✓	✓	
PT		✓	✓			✓
GP	✓	✓	✓	✓	✓	✓
JK			✓	✓	✓	
DK				✓		✓
AS		✓	✓	✓	✓	
KSr		✓	✓	✓	✓	✓

**Question: What topics in the mathematical curriculum (and in which grades) are relevant for functional thinking development? Justify your answer.**

The following tables show the topics that the following pre-service teachers mentioned as a topic suitable for developing functional thinking:

PRE TEST	KH	SP	KS	IO	PT	GP	JK	DK	AS	KSr	
Different classes of functions and their properties	✓	✓	✓				✓	✓	✓	✓	7
Trigonometry	✓										1
Proportionality			✓								1
Word problems						✓	✓				2
Patterning											0
Equations		✓					✓				2
Algebraic expressions								✓			1
Coordinate system, number line							✓				1
Arithmetic operations (Addition, ...)							✓				1
Negative numbers							✓				1
Sequences											0
Statistics			✓								1
Derivative of the function			✓								1
Reading graphs											0
Speed, time, distance		✓									1
Optimisation tasks				✓							1
Operations with fractions											0
Geometry					✓	✓					2
Secondary school (from 9 grade)	✓	✓					✓	✓	✓	✓	6
Lower secondary school (grades 7-8)	✓	✓							✓		3
Primary school below 7 grade		✓			✓			✓			3
Kindergarten											0
Total	4	6	4	1	2	2	7	4	3	2	

The post test revealed a slightly different approach:

POST TEST	K H	SP	KS	IO	PT	G P	JK	D K	AS	KS r	
Different classes of functions and their properties	✓	✓	✓				✓	✓	✓	✓	7
Proportionality	✓	✓					✓		✓		4
Word problems	✓			✓			✓				3
Patterning	✓				✓						2
Algebraic expressions											0
Coordinate system, number line						✓		✓			2
Arithmetic operations (Addition, ...)						✓					1
Sequences			✓								1
Statistics									✓		1
Reading graphs			✓	✓		✓	✓				4
Speed, time, distance		✓							✓		2
Optimisation tasks				✓							1
Operations with fractions					✓						1
Geometry					✓						1
Secondary school (from 9 grade)		✓	✓				✓	✓	✓	✓	6
Lower secondary school (grades 7-8)		✓	✓	✓	✓	✓		✓	✓	✓	8
Primary school below 7 grade		✓		✓	✓	✓	✓	✓	✓		6
Kindergarten						✓					1
Total:	4	6	5	5	5	6	5	5	7	3	

In both the pre-test and the post-test, the pre-service teachers talked about the different levels at which functional thinking can be developed. They naturally focused on the unit on functions and relationships, often mentioning direct or indirect proportionality.

The post-test highlighted the position of grades 7-8 (pre: n=3, post: n=8), where students can learn about function at pre-definition stage. There was a significant increase in the number of statements concerning the development of functional thinking in the younger grades of primary school and even at kindergarten level (pre: n=3, post: n=7).

**Question: What do you consider to be a goal of teaching about functions? Expand your answer.**

Goals which were mentioned in the tests are described in the following table:

	Pre-test	Post-test
Understanding / describing dependencies and relationships	7	7
Important because of real-life contexts	2	5
Functional thinking development	0	7
Understanding functions	3	6
Representations	1	5
Prediction	1	2
General goals	5	3

### Likert questionnaire of course assessment (11 items)

The table below shows for each item the average rating provided by the course participants. The results are very good and show that the course has served its purpose.

	Mean
1. I learned interesting things in the teacher training course about mathematics.	4,7
2. I learned interesting things in the teacher training course about teaching.	4,6
3. The knowledge I gained is useful for teaching Mathematics.	4,8
4. The structure of the teacher training course was appropriate and effective.	4,4
5. The content of the teacher training course was appropriate and effective.	4,6
6. The way of delivery of the teacher training course was appropriate and effective.	4,6
7. I will use the material of the project in my teaching.	4,3
8. I will use the digital tools of the project in my teaching.	4,5
9. The digital material of the project is interesting.	4,7
10. The digital material of the project facilitates conceptual understanding of the mathematics concepts.	4,8
11. I would recommend this seminar to a colleague of mine.	4,7

### Question: What did you like best about this teacher training course?

The following are all the responses to this question from course participants:

*PST1 - Possibility to repeat knowledge, - examples of interesting tasks, - presentation of approaches to teaching about functions, - interesting theoretical aspects*

*PST2 - Choosing different tasks; - Reflecting on which tasks develop what and what their purpose is; - digital materials e.g. turtle, - atmosphere, - working in groups, - interesting, non-obvious tasks appeared.*

*PST3 What I liked most is that we learned about a lot of interesting tools (mostly digital) that we can use in our future work.*

*PST4 What I liked most was that I was able to look at functions in a different way and share my insights with the other people taking this course. In addition, I liked the assignments set before us, through which we were able to learn a lot, and the multitude of ways of teaching about functions presented by the course leader and the multitude of tools we will be able to use when teaching about functions.*

*PST5 We were able to extend our knowledge of functions, noticing how important and useful the concept of function is in everyday life. We observed the function not only as a task solver, but through the experiments carried out, the function was better understood.*

*PST6 Demonstration of different, interesting ways to teach a lesson (Number lines, digital aids).*

*PST7 Showing new ways to shape functional thinking, identifying and explaining important topics in function topics.*

*PST 8 - Many references to real-life situations that can also be referred to in the lesson, - Demonstration of activities that can be used in a mathematics lesson with students one day, which develop students' functional thinking, - Demonstration of many teaching aids (mainly in electronic version) that can be used with students.*

*PST9 - Showing how to transfer the understanding of functions into aspects of everyday life and where we can apply them. - Presenting functions in an interesting way, e.g., using movement.*

*PST10 The format of the course was enjoyable. Due to the fact that we have a lot of pure mathematics and theory this semester, this course allowed us to relax and I was able to redirect my thoughts on the didactic path. The presentations, the aspects covered, the assignments and the meeting with the Physicist was very developing.*

**Question: What suggestions do you have for improving this teacher training course?**

The following are all the responses to this question from course participants:

*PST1 A bit too many tasks at the beginning of the course (they were interesting, but it took a long time), improvement of the translations*

*PST2 Improving the quality of materials - translation and print.*

*PST3 I think an improvement would be to spend more time on it and to improve the translation of the material.*

*PST4 I think it would be more possible to interweave static and active/movement elements of activities.*

*PST5 I really enjoyed the classes, I would choose this course again. The course was very interesting.*

*PST6 More hours so that even more practical things can be seen.*

*PST7 More efficient discussion of prepared tasks (although each topic was important)*

*PST8 I enjoyed the practical exercises the most, so maybe even more of them. However, I still like the course as it is.*

*PST9 Improved organisation of time spent working on specific issues/tasks.*

*PST10 It would be nice if there were more courses like this, but concerning more than just functional thinking. On the practical side I wouldn't change anything, the classes were conducted in an interesting way.*

**Question: What other suggestions would you like to share with us regarding this teacher training course?**

The following are all the responses to this question from course participants:

*PST1 Very interesting.*

*PST2 No other comments.*

*PST3 I think this course could have been better published so that more future, as well as in-service teachers could have taken part in in.*

*PST4 I have no further suggestions. I think this course is very necessary and useful for future teachers.*

*PST5 There could be even more experiences.*

*PST6 Some of the material, especially the printed material, was not of good quality.*

*PST7 -*

*PST8 I have no other suggestions.*

*PST9 -*

*PST10 It could be nice to be able to take part in some kind of activity at school as part of the course that would address this issue.*

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#### Discussion:

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We can see that the pre-service teachers increased their pedagogical content knowledge and their content knowledge.

The pre-service teachers have revealed particularly spectacular progress in their understanding of what functional thinking is and in their understanding of all aspects of the concept of function. Concerning the question on topics in the mathematical curriculum and their grades which are relevant for functional thinking development, we also observe a positive development. In the area of defining goals of teaching about functions we can also observe a progress i.e. increasing number of answers in almost each category and especially the post-test highlighted the position of grades 7-8 and significant increase role of development of functional thinking in the younger grades of primary school and even at kindergarten level. This was an important course goal, because of introducing the definition of function in Polish curriculum only in grade 9. Therefore, pre-service teachers have learnt how to develop functional thinking at pre-definition stage.

Pre-service teachers were very active, they enjoyed the activities and cooperated willingly in small groups.

The detailed evaluation of the course presented above is very positive and the pre-service teachers' suggestions that the course should be longer and that similar courses should take place in other subject areas is a convincing argument for its effectiveness. **It is therefore most significant that the Mathematics Department at the Pedagogical University of Krakow has decided to introduce the course, "Developing Functional Thinking in Students at Different Educational Stages", as a permanent 20-hours course in the study plan for pre-service mathematics teachers at 4<sup>th</sup> year of their master studies.**

Moreover the lessons we have learnt and will use in the next rounds of the course are as follows:

First, we need to be aware of the language issue in Polish, where "myślenie funkcyjne" (functional thinking) literally means "thinking that works well, is effective or may be applied". This issue needs to be addressed directly with the pre-service teachers.

Secondly, our pre-service teachers' previous experience of learning and teaching is mostly 'traditional', quite far from the constructivist way. Therefore, their belief systems seemed to be in conflict with the design principles of learning environments. To make the course more effective, their belief systems will be addressed in a more direct way.