Co-funded by the Erasmus+ Programme of the European Union

Report on testing learning environments: Walking Graphs, Temperature and Filling Vessels

Module:	Walking Graphs Temperature Filling vessels
Responsible Partner:	Ludwigsburg University of Education, Germany
Grade Level/Age Range:	Grade 8, 13-14 years
Sample size:	28 students
Brief Description of Testing / Intervention:	The three learning environments (5 lessons, 45 min each) were implemented during the unit of functional relationships and linear functions in grade 8. In total, the unit lasted about 5 weeks. All lessons, not only those of the three learning environments, were designed with the four design principles of the FunThink project in mind (inquiry-based learning, situatedness, embodiment and digital tool) but not all lessons included all design principles. The pre- and post-test were completed prior to the first and after the last lesson of the unit. Walking graphs: Students experience distance-time scenarios physically and digitally and develop a qualitative understanding of graphs and covariational reasoning. Temperature: The learning environment focuses on the properties of functions. Students develop an understanding for what is and what is not a function. Different to the proposed teacher guide, some tasks could only be completed in the whole class and not individually due to technical issues. Filling vessels: Similar to walking graphs, this learning environments supports the qualitative understanding of graphs and the development of covariational reasoning. During this intervention, only the digital part of the learning environment was used.

Method:

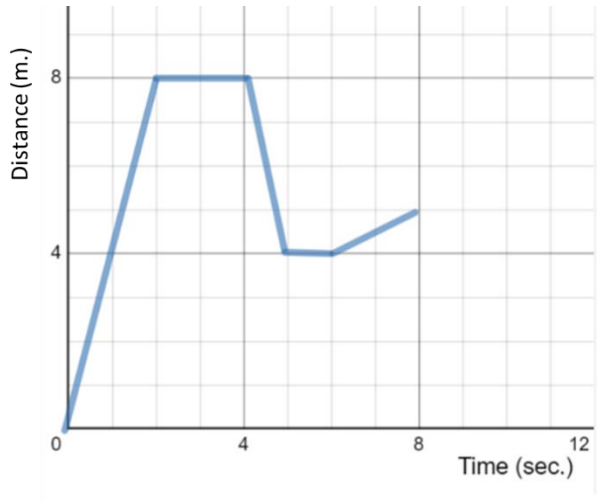
This intervention was carried out with 28 grade 8 students of a middle achieving secondary school. In the previous school year, the students already learned about proportional relationships but functions themselves were not introduced prior to this intervention. The intervention took place in early 2023. The classes were taught by a researcher involved in the FunThink project and the development of the learning environments. Participation in the testing was voluntary and had no influence on students' grades in their mathematics class. Consent was collected prior to the pre-test. The described results and items were part of the pre- and post-test which was completed by the students prior to and after the intervention.

General item 1 is based on an item used by Duijzer (2020) and measures graph interpretation and graph construction. Students investigate a graph with data about a travelling remote car

(distance-time). The first question requires global and local interpretation of the graph as students have to identify which parts of the graph represent moving away or moving towards a person. The second question asks to identify when the car moves the fastest. The third question asks to extend the graph for the following seconds based on a given description.

General Item 1:

Ann plays with a remote-control car toy. The following graph presents the distance of the car from Ann in respect to time.



a. When was the car moving away from Ann and when towards Ann? Please explain.

b. When did the car move the fastest? Please explain.

c. Complete the graph for the next four seconds based on the following:

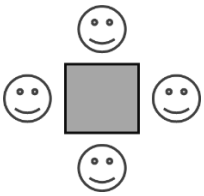
> "The car moved away from Ann for another one second and then moved towards her, without reaching her."

General item 2 is based on the Birthday Party item used by Blanton et al. (2015). The first two questions of the item require to find a term of a pattern that could be calculated based on a recursive, covariation or correspondence rule. The third part of this item asks for a general rule of the used pattern.

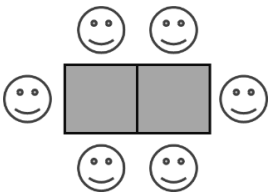
General Item 2:

Brady is having his friends over for a birthday party. He wants to make sure he has a seat for everyone. He has square tables.

He can seat 4 people at one square table in the following way:



If he joins another square table to the first one, he can seat 6 people:



a. If Brady has 8 tables, how many people can he seat at his birthday party?

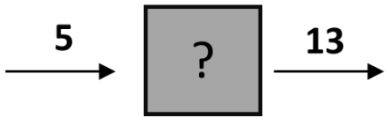
b. And how about 20 tables?

c. Can you find a rule that describes the relationship between the number of tables and the number of people who can sit at the tables?

General item 3 was developed for the purpose of the study based on ideas suggested by Pittalis et al. (2020) and Ng (2018) and measurs students' ability to identify the numerical relation between two sets of values to find the input or output value of a function machine. Students were also asked to express the rule of the machine.

General Item 3:

1. Find below a function machine. A number is entered, and the machine gives an output value based on a secret rule.



The table shows some inputs and outputs of this machine. Complete the empty cells.

INPUT	**OUTPUT**
0	3
5	13
7	17
10	23
12	
15	
	11
	43

2. John entered the symbol * in the machine. What will be the output? Please explain.

Results and Discussion:

General Item 1:

In the following, the results of this item will be highlighted for each part separately.

Part a: This part was coded for correctness and the type of reasoning used. Table 1 shows the answers provided by the students in terms of correctness. It can be seen that the amount of wrong answers decreased from the pre- to the post-test. As a result, the partly correct and correct answers increased in the post-test.

Table 1: Correctness general item 1 part a

	Wrong answer	Partly correct answer	Correct answer	No answer	Total
Pre-test	6	10	11	1	28
Post-test	1	12	13	2	28

With regard to the observed reasoning, more than half of the students referred in their answers to a single variable in both tests. This way of reasoning only slightly decreased from the pre- to the post-test. On the other hand, in the post-test, considerably more students indicated their ways of reasoning and showed multi variable reasoning.

Table 2: Reasoning general item 1 part a

	Single variable reasoning	Multi variable reasoning (2 variables)	Multi variable reasoning (3 variables)	No reasoning visible	Total
Pre-test	18	3	0	7	28
Post-test	16	8	2	2	28

Part b: Similar to part a, this part was also coded for correctness and the type of reasoning used. Table 3 indicates the correctness of the students' answers and table 4 the used reasoning. It can be seen that more students were able to identify both intervals correctly in the post-test and that the reasoning shifted for some students from single variable to multi variable reasoning.

Table 3: Correctness general item 1 part b

	Wrong answer	One interval correct	Both intervals correct	No answer	Total
Pre-test	5	18	3	2	28
Post-test	4	14	7	3	28

Table 4: Reasoning general item 1 part a

	Single variable reasoning	Multi variable reasoning (2 variables)	Multi variable reasoning (3 variables)	No reasoning visible	Total
Pre-test	10	11	1	6	28
Post-test	5	12	5	6	28

Part c: In this part of the general item 1, students were asked to continue the given graph according to a concrete description. This part was only coded for correctness. Table 5 indicates that no wrong answer was provided in the pre-test nor in the post-test. A shift from

partly correct answers to completely correct answers could be observed for several students from pre- to post-test.

Table 5: Correctness general item 1 part c

	Wrong answer	Partly correct answer	Correct answer	No answer	Total
Pre-test	0	8	16	4	28
Post-test	0	1	24	3	28

In total, an improvement in the students' answers could be observed for this item. This indicates that the graph interpretation and graph construction skills which are important for functional thinking improved during the intervention.

Results General Item 2:

The results for part a and b (number of children for a given number of tables) of this item were coded in combination. Table 6 indicates the answers provided for these two parts, whereas Table 7 shows results with regard to the correctness of the provided rule (part c). A partly correct answer indicates that either part a or part b were answered correctly. A slight increase in correct answers and a decrease in wrong rules can be observed for this item after the intervention. In the post-test, many students only wrote down the correct answers for part a and b but did not indicate any rule on how they received the correct answers. This can be seen in the increase in the category *no rule* in table 7. The category *other* in table 7 indicates rules that do not focus on the relation between tables and people but that are still correct. For this item, it has to be noted that some students included other combinations of tables in their answers. For this analysis, other combinations were not taken into consideration and coded as wrong answers. This differentiation will be part of a deepening analysis. Even though this was not being considered, a slight increase in the students' skill required in this task could be observed.

Table 6: Correctness general item 2

	Wrong answer	Partly correct answer	Correct answer	No answer	Total
Pre-test	8	6	12	2	28
Post-test	8	4	14	2	28

Table 7: Correctness rule general item 2

	Wrong rule	Correct rule	No rule indicated	Other	Total
Pre-test	12	8	8	0	28
Post-test	8	6	12	2	28

Results General Item 3:

The results for general item 3 show that only half of the students in the pre- and in the post-test completed this item. In addition, the correct answers decreased from pre- to post-test. Possible explanations for these findings could be that this kind of task is not commonly used in German schools and the used intervention did not include corresponding tasks. Further investigation is needed to understand the changes and results of this item. With regard to the

used reasoning, students mostly referred to a correspondence view, in particular in the post-test.

Table 8: Correctness table general item 3

		Wrong answer	Partly correct answer	Correct answer	No answer	Total
Pre-test	Output	3	1	10	14	28
Post-test		5	2	7	14	28
Pre-test	Input	3	1	9	15	28
Post-test		4	4	6	14	28

Table 9: Reasoning general item 3

	Covariational reasoning	Correspondence reasoning	Other correct strategy	Wrong strategy	no answer
Pre-test	2	6	2	1	17
Post-test	0	7	0	1	20

Discussion

The results for general items 1 and 2 indicate a positive effect on the student abilities with regard to functional thinking. Skills related to interpreting and constructing graphs, as well as skills needed for analysing and continuing patterns were further developed during the intervention. All of these skills are important for developing functional thinking. Therefore, the intervention including the learning environments walking graphs, temperature and filling vessels can contribute to a positive effect on the students' functional thinking. The results of item 3 need further investigation and possible adaptions of the intervention need to be considered.

In conclusion, the results of the intervention are promising and seem to contribute to the development of students' functional thinking. Further research will allow deeper insights into the development of students' functional thinking and will follow in a next step.