Report on testing learning environment: Nomogram (upper secondary: function composition)

Module:	Function composition through nomograms (upper secondary)
Responsible Partner:	Utrecht University
Grade Level/Age Range:	Grades 10 and 11 (ages 14-17)
Sample size:	40
Brief Description of Testing / Intervention:	**Introduction:** In the Funthink project we have various function representations i.e., graph, table, formula. Each representation has its usefulness for highlighting the various aspects of functional thinking. For example, graphs are useful to highlight covariation, tables are useful to highlight correspondence and formulas are useful to highlight input output. Of course, the usefulness is not exclusive. Graphs, for example, may also prove useful for objectification of a function. The current learning environment focusses on yet another representation of a function namely the nomogram. The nomogram consists of two number lines. The function relation between the independent and the dependent variable is expressed by arrows pointing from the independent number line to the dependent number line. The nomogram is particularly useful for highlighting the input-output and the object aspects. Specifically: the composition of functions can be demonstrated concisely by composing two nomograms, where the dependent number line of the first nomogram is, in turn, the independent number line of the second nomogram. This brings us to the learning goal of this learning environment: **Learning goal:** The student can demonstrate comprehension of function composition by executing a computation based on the interpretation of a composition of nomograms.

Method:

Three classes of high achieving students in grades 10 and 11 (ages 14-17) participated ($n = 40$). The students were given a pretest prior to the intervention and a after the intervention posttest (see appendix). The intervention consisted of a 60-minute, teacher-guided interaction with the learning environment.

After data collection from pre- and posttest only the data of students demonstratable present during all three events (pretest, intervention and posttest) were selected. ($n = 32$ after selection). After the students completed the pretest, prior to a thorough analysis, it was clear that all students performed maximally on the second test item in the pretest, it was therefore removed from the posttest.

Results and Discussion:

Results

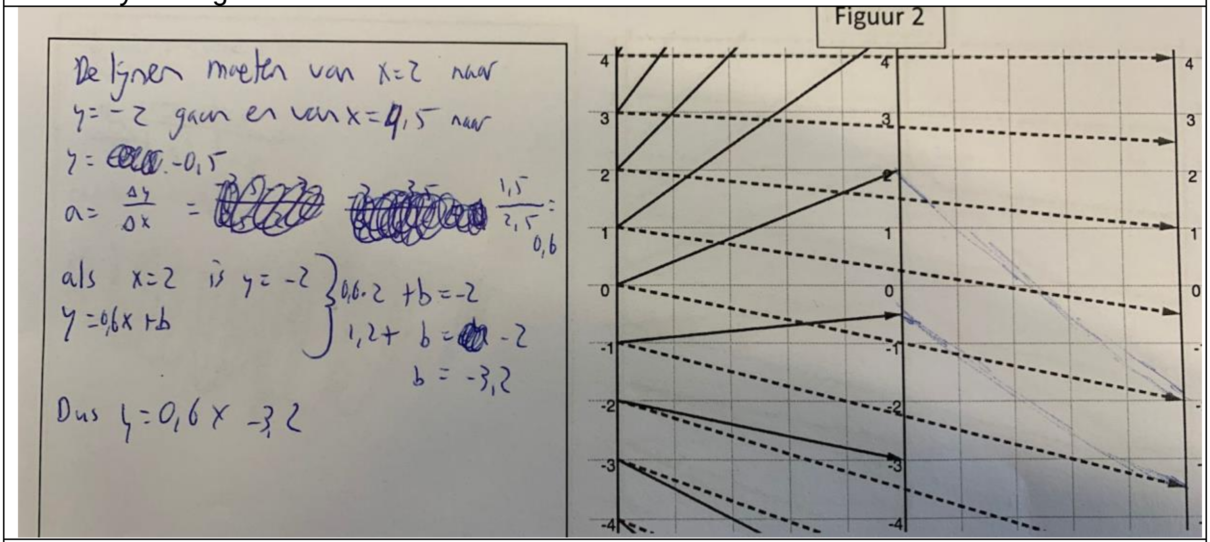
For pre- and posttest, the proportion of students that submitted a correct response is presented in Table 1. The second item in the table refers to the item of the pretest and posttest such that they correspond.

Item	Pretest proportion correct	Posttest proportion correct
1	.84	.84
Pretest 3 <-> Posttest 2	.28	.44

Table 1: pre- and posttest results

We found evidence of the interpretation of the nomogram, for instance in this excerpt:

Given the functions $f(x) = 2{,}5x + 2$ and $h(x) = 1{,}5x - 2$
Find a Formula $g(x) = ax + b$ such that $g\big(f(x)\big) = h(x)$.
This means $g(x) = a \cdot (2{,}5x + 2) + b = 1{,}5x - 2$
You may use figure 2.



The student uses the nomogram to find two pairs of values that satisfy the function relation for g. These values are then used to find a formula for g.

Discussion

From Table 1 we clearly see a ceiling effect concerning test item 1. For item 2 however, a standard z-test for the proportion confirms significant learning gain.

$$z = \frac{p_0 - \hat{p}}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} = 1.823 \text{ and } P(Z > 1.823) = .034$$

Where: $H_0\!: p = .28$ against $H_1\!: p > .28$ with $\hat{p} = .44$ and $n = 32$.

We may argue that the learning goal is achieved when a student completes task 2. That is, the student solves a functional equation $g\big(f(x)\big) = h(x)$ $(*)$ for a linear function g, when linear f and h are given. The solution can be obtained by completing a partial composition nomogram where f and h are given as nomograms. We argue that, to a student, a solution to an equation is an object-like entity. Therefore, the learning environment contributes to the understanding of a function as an object. Furthermore, students can use the nomogram to obtain the solution f to the functional equation $(*)$. They do so by noticing that the input of g coincides with the output f while the output of g coincides with the output of h. Therefore, we can argue that the learning environment contributes to the understanding of the input-output aspect of a function.

Appendix

Pretest

1) Look at figure 1. The left number line represents the x-axes and the right number line represents y-axes.

Find a formula for the relation between x and y.

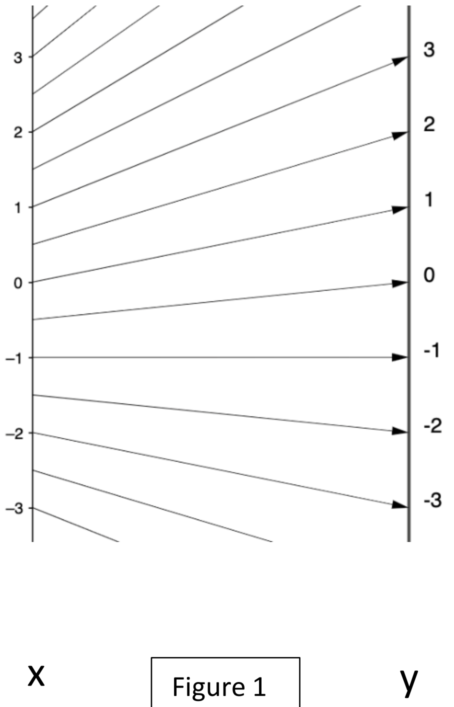


Figure 1

2) Draw arrows in figure 2 corresponding to the formula:

$y = x + 1$

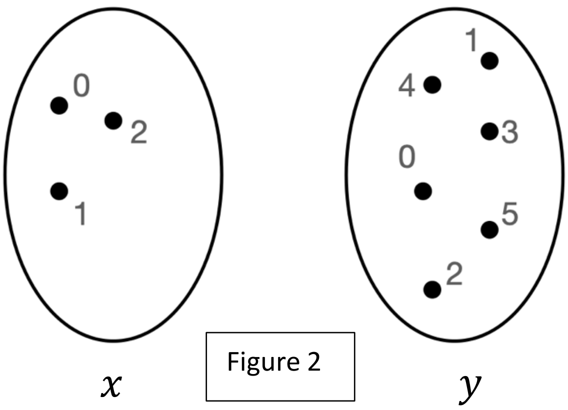


Figure 2

3) Given the functions $f(x) = 2x + 2$ and $h(x) = x - 1$

Find a Formula $g(x) = ax + b$ such that $g\big(f(x)\big) = h(x)$.

This means: $g(x) = a \cdot (2x + 2) + b = x - 1$

You may use figure 3.

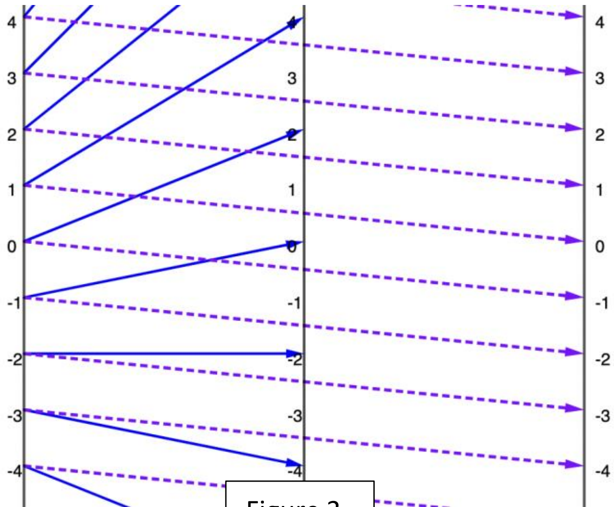


Figure 3

Pretest Key

1)
$(0,1)$ satisfies the function relation 1p
$y = 2x + 1$ 1p

2) Connecting the points 1p

3) Indicating usage of two lines of the nomogram of g 1p
Reading two points that satisfy g from the lines 1p
Calculating a 1p
Calculating b 1p

Or:

$a \cdot 2x + 2a + b = x - 1$ 1p

$2a = 1$ 1p

$2a + b = -1$ 1p

$a = 0{,}5$ and $b = -2$ 1p

Above steps may be implicit in student answer

Posttest

1) See figure 1. The left number line represents the x-axis and the right number line represents the y-axis. Give a formula for the relationship between $x$and y.

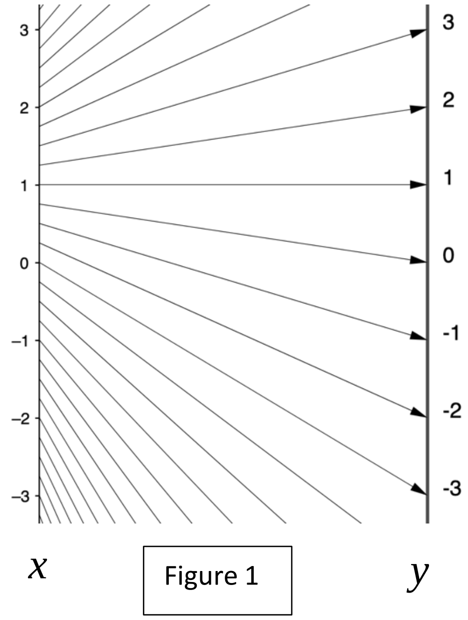


Figure 1

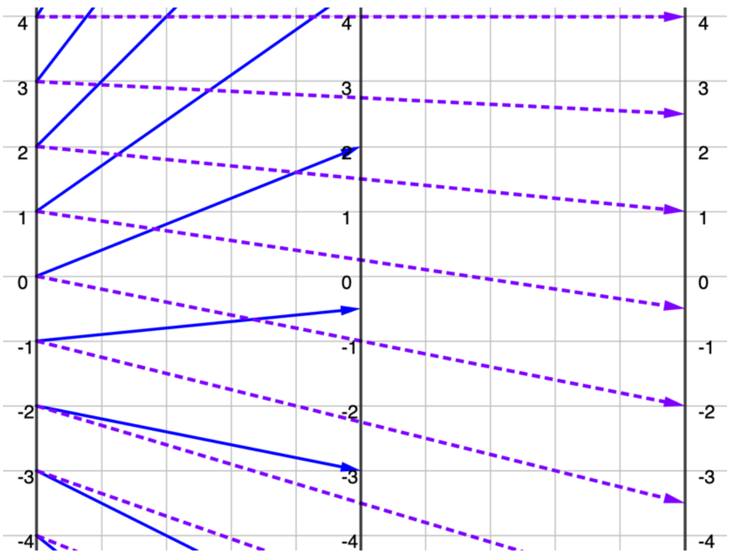
2) Given are the functions $f(x) = 2{,}5x + 2$ and $h(x) = 1{,}5x - 2$

Give a formula $g(x) = ax + b$such that $g\big(f(x)\big) = h(x)$.

This means: $g(x) = a \cdot (2{,}5x + 2) + b = 1{,}5x - 2$

You can use Figure 2 for this.

Figure 2



Posttest Key

1)

$(0,-3)$satisfies the function relation	1p
$y = 4x - 3$	1p

2)

Indicating usage of two lines of the nomogram of g	1p
Reading two points that satisfy g from the lines	1p
Calculating a	1p
Calculating b	1p

Or:

$a \cdot 2{,}5x + 2a \ + b = 1{,}5x - 2$	1p
$2{,}5a = 1{,}5$	1p
$2a + b = -2$	1p
$a = 0{,}6$ and $b = -3{,}2$	1p

Above steps may be implicit in student answer